2.1 Quadratic Functions

We have four methods to solve quadratic equations . . .

1) **Factoring**
2) **Quadratic Formula**
3) **Completing the Square**
4) **Graphing** (We will be using <u>Vertex Form</u>)

Solve $3x^2 + x - 2 = 0$

Method 1: <u>Factoring</u>

$3x^2 + x - 2 = 0$

$(3x-2)(x+1)=0$

$3x-2=0 \qquad x+1=0$

$\boxed{x=\frac{2}{3}} \qquad \boxed{x=-1}$

Method 2: **Quadratic Formula**

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$3x^2 + x - 2 = 0$

$$x = \frac{-1 \pm \sqrt{1^2-4(3)(-2)}}{2(3)}$$

$$x = \frac{-1 \pm \sqrt{25}}{6}$$

$$x = \frac{-1 \pm 5}{6}$$

$$\boxed{x = \frac{2}{3} \text{ and } -1}$$

The Discriminant: Allows us to determine how many roots we have and what type of roots they are.

$\boxed{b^2 - 4ac}$

$b^2 - 4ac < 0$	The equation has 2 imaginary roots
$b^2 - 4ac = 0$	The equation has 1 (double) real root
$b^2 - 4ac > 0$	The equation has 2 real roots

- If $b^2 - 4ac$ is a perfect square there will be 2 rational roots
- If $b^2 - 4ac$ is NOT a perfect square, there will be 2 irrational roots

Method 3: Completing the Square

$$3x^2 + x - 2 = 0$$

Note: "a" must be "1"

$$\frac{3x^2}{3} + \frac{x}{3} - \frac{2}{3} = \frac{0}{3}$$

$$x^2 + \frac{1}{3}x - \frac{2}{3} = 0$$

$$x^2 + \frac{1}{3}x + \_\_\_\_ = \frac{2}{3} + \_\_\_\_$$

Take half of "b" and square it. Add that to both sides

$$x^2 + \frac{1}{3}x + \left(\frac{1}{6}\right)^2 = \frac{2}{3} + \left(\frac{1}{6}\right)^2$$

$$\left(x+\frac{1}{6}\right)\left(x+\frac{1}{6}\right) = \frac{2}{3} + \frac{1}{36}$$

$$\left(x+\frac{1}{6}\right)^2 = \frac{25}{36}$$

$$x+\frac{1}{6} = \pm\sqrt{\frac{25}{36}}$$

$$x+\frac{1}{6} = \pm\frac{5}{6}$$

$$x = -\frac{1}{6} \pm \frac{5}{6}$$

$$x = \frac{5}{6} - \frac{1}{6} = \boxed{\frac{2}{3}}$$

$$x = \frac{-5}{6} - \frac{1}{6} = \boxed{-1}$$

Method 4: <u>Graphing</u>

Write the quadratic function in vertex form and sketch its graph. Identify the vertex, axis of symmetry, and x-intercepts.

Ex. $f(x) = 2x^2 + 8x + 7$

<u>Vertex Form</u>: $f(x) = a(x - h)^2 + K$

<u>Vertex:</u> (h, k)

<u>Axis of symmetry</u>: the line x = h

<u>To find x- intercepts</u>: Set f(x)=0 and solve for "x"

First put $f(x) = 2x^2 + 8x + 7$ in <u>Vertex Form</u> . . .

$f(x) = (2x^2 + 8x + \_\_\_) + 7 - \_\_\_$

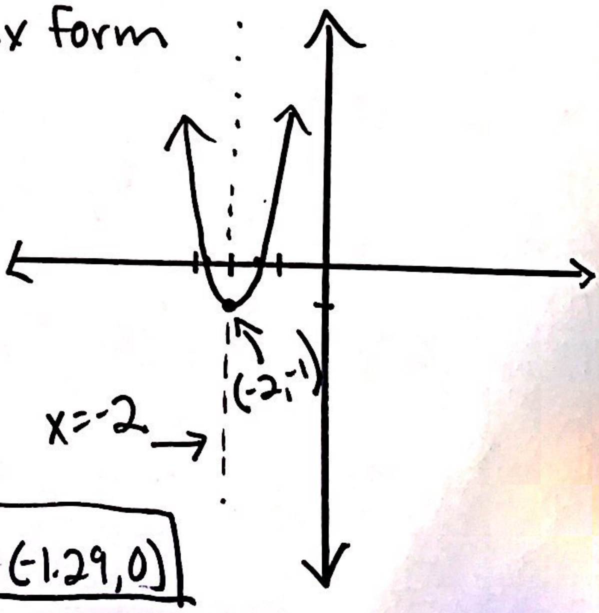
Need to Make a "1"

$f(x) = \boxed{2}(x^2 + 4x + \underline{2^2}) + 7 - \boxed{2}(2^2)$

$f(x) = 2(x^2 + 4x + 4) + 7 - 8$

$f(x) = 2(x+2)(x+2) - 1$

$\boxed{f(x) = 2(x+2)^2 - 1}$ Vertex form



Vertex: $(-2, -1)$

Axis of Symmetry: $x = -2$

To find x-intercepts . . .

$2x^2 + 8x + 7 = 0$

$x = \dfrac{-8 \pm \sqrt{8^2 - 4(2)(7)}}{2(2)}$

$x \approx -2.7, -1.29 \rightarrow \boxed{(-2.7, 0) \;\&\; (-1.29, 0)}$

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Write the vertex form of the quadratic function whose graph is the parabola with vertex (1, 2) and that passes through the point (3, -6).

"h" "k"

Vertex: (1, 2)

Start with...

$f(x) = \underline{a}(x-\underline{h})^2 + \underline{k}$

$\downarrow$

$y = a(x-1)^2 + 2$

Substitute in (3, -6)

$-6 = a(3-1)^2 + 2$

$-6 = 4a + 2$

$-8 = 4a$

$\boxed{a = -2} \longrightarrow \boxed{f(x) = \underline{-2}(x-\underline{1})^2 + \underline{2}}$

Example:

A ball follows $h(t) = -16t^2 + 80t + 3.5$. Describe the # and type of roots. How long will the catcher have until the ball hits the ground?

$$b^2 - 4ac$$

$$= 80^2 - 4(-16)(3.5)$$

$$= 6624 \rightarrow \text{2 Real Irrational Roots}$$

To find how long until the ball hits the ground...

Set $h(t) = 0$

$$-16t^2 + 80t + 3.5 = 0$$

$$\downarrow$$

$$t = \boxed{5.04}, -0.04$$