

# Algebra Review

## EXAMPLE 1

### Determining Whether a Point is on a Line

Decide whether  $(3, -2)$  is a solution of the equation  $y = 2x - 8$ .

$$-2 = 2(3) - 8 \quad \text{Substitute 3 for } x \text{ and } -2 \text{ for } y.$$

$$-2 = -2 \quad \text{Simplify.}$$

The statement is true, so  $(3, -2)$  is a solution of the equation  $y = 2x - 8$ .

## EXERCISES

Decide whether the given ordered pair is a solution of the equation.

- |  |                                     |
|--|-------------------------------------|
| 1. $y = 6x + 4; (-2, 8)$               | 2. $y = -10x - 2; (1, -12)$         |
| 3. $y = -\frac{1}{4}x - 18; (-4, -17)$ | 4. $y = \frac{3}{2}x + 10; (4, 12)$ |
| 5. $y = \frac{5}{9}x + 34; (-9, 27)$   | 6. $y = \frac{2}{3}x - 6; (9, 0)$   |
| 7. $y = \frac{4}{5}x - 2; (10, -3)$    | 8. $y = \frac{1}{2}x + 7; (4, 7)$   |
| 9. $2x - 3y = 10; (3, 4)$              | 10. $9x - y = -4; (-1, -5)$         |
| 11. $y - 6 = \frac{3}{4}x; (8, 12)$    | 12. $y + 5 = \frac{5}{3}x; (9, 10)$ |

## EXAMPLE 2

### Calculating Slope

Find the slope of a line passing through  $(3, -9)$  and  $(2, -1)$ .

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Formula for slope}$$

$$m = \frac{-1 - (-9)}{2 - 3} = \frac{-1 + 9}{-1} \quad \text{Substitute values and simplify.}$$

$$m = \frac{8}{-1} = -8 \quad \text{Slope is } -8.$$

## EXERCISES

Find the slope of the line that contains the points.

- |                         |                          |                          |
|-------------------------|--------------------------|--------------------------|
| 13. $(4, 1), (3, 6)$    | 14. $(-8, 0), (5, -2)$   | 15. $(5, 6), (9, 8)$     |
| 16. $(0, -4), (7, -3)$  | 17. $(-1, 7), (-3, 18)$  | 18. $(-6, -4), (1, 10)$  |
| 19. $(4, -10), (-2, 2)$ | 20. $(11, 1), (-11, 1)$  | 21. $(14, -5), (5, 8)$   |
| 22. $(-7, 5), (-1, -1)$ | 23. $(-12, 8), (-3, -6)$ | 24. $(-9, 13), (2, -10)$ |
| 25. $(12, 3), (0, -4)$  | 26. $(9, -8), (-7, 10)$  | 27. $(2, -5), (6, -6)$   |

# Algebra Review

2

## **EXAMPLE 3** Finding the Equation of a Line

Find an equation of the line that passes through the point (3, 4) and has a y-intercept of 5.

$$y = mx + b \quad \text{Write the slope-intercept form.}$$

$$4 = 3m + 5 \quad \text{Substitute 5 for } b, 3 \text{ for } x, \text{ and 4 for } y.$$

$$-1 = 3m \quad \text{Subtract 5 from each side.}$$

$$-\frac{1}{3} = m \quad \text{Divide each side by 3.}$$

The slope is  $m = -\frac{1}{3}$ . The equation of the line is  $y = -\frac{1}{3}x + 5$ .

### EXERCISES

Write the equation of the line that passes through the given point and has the given y-intercept.

28. (2, 1);  $b = 5$

29. (-5, 3);  $b = -12$

30. (-3, 10);  $b = 8$

31. (7, 0);  $b = 13$

32. (-3, -3);  $b = -2$

33. (-1, 4);  $b = -8$

34. (-11, 8);  $b = -14$

35. (4, -6);  $b = -2$

36. (5, -8);  $b = 7$

37. (-2, -1);  $b = -5$

38. (2, 3);  $b = 2$

39. (3, 0.5);  $b = 1.5$

## **EXAMPLE 4** Finding the Equation of a Line

Write an equation of the line that passes through the points (4, 8) and (3, 1).

Find the slope of the line.

$$m = \frac{1 - 8}{3 - 4} \quad \text{Substitute values.}$$

$$m = \frac{-7}{-1} = 7 \quad \text{Simplify.}$$

$$1 = 7(3) + b \quad \text{Substitute values into } y = mx + b.$$

$$1 = 21 + b \quad \text{Multiply.}$$

$$-20 = b \quad \text{Solve for } b.$$

The equation of the line is  $y = 7x - 20$ .

### EXERCISES

Write an equation of the that passes through the given points.

40. (6, -3), (1, 2)

41. (-7, 9), (-5, 3)

42. (5, -1), (4, -5)

43. (-2, 4), (3, -6)

44. (-3, -7), (0, 8)

45. (1, 2), (-1, -4)

46. (6, -2), (0, 4)

47. (-4, 3), (-3, -3)

48. (-3, 2), (-5, -2)

49. (10, -9), (14, -1)

50. (-1, -2), (5, 0)

51. (-6, 4), (6, -1)

# Algebra Review

## EXAMPLE 1 Distance Formula

Find the distance between the points  $(-4, 3)$  and  $(-7, 8)$ .

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-7 - (-4))^2 + (8 - 3)^2} \\ &= \sqrt{(-3)^2 + (5)^2} \\ &= \sqrt{34} \end{aligned}$$

## EXERCISES

Find the distance between the points.

1.  $(3, 6), (0, -2)$
2.  $(5, -2), (-6, 5)$
3.  $(-3, 4), (1, 4)$
4.  $(-6, -6), (-3, -2)$
5.  $(8, -2), (-3, -6)$
6.  $(-8, 5), (-1, 1)$

## EXAMPLE 2 Combining Like Terms

Simplify.

$$8x^2 + 16xy - 3x^2 + 3xy - 3x$$

$$8x^2 - 3x^2 + 16xy + 3xy - 3x$$

$$5x^2 - 3x + 19xy$$

Group like terms.

Simplify.

## EXERCISES

Simplify.

7.  $6x + 11y - 4x + y$
8.  $-5m + 3q + 4m - q$
9.  $-3p - 4t - 5t - 2p$
10.  $9x - 22y + 18x - 3y$
11.  $3x^2y - 5xy^2 + 6x^2y$
12.  $5x^2 + 2xy - 7x^2 + xy$

## EXAMPLE 3 Solving Equations with Variables on Both Sides

Solve.

$$6a - 12 = 5a + 9$$

$$a - 12 = 9$$

$$a = 21$$

Subtract  $5a$  from each side.

Add 12 to each side.

## EXERCISES

Solve the equation.

13.  $3x + 5 = 2x + 11$
14.  $-14 + 3a = 10 - a$
15.  $8m + 1 = 7m - 9$
16.  $y - 18 = 6y + 7$
17.  $2s + 1 = 7s + 1$
18.  $3a - 12 = -6a - 12$
19.  $-2t + 10 = -t$
20.  $11q - 6 = 3q + 8q$
21.  $-7x + 7 = 2x - 11$

# Algebra Review

4

## EXAMPLE 4 Solving Inequalities

Solve.

a.  $5x - 4 \geq 4x + 6$

b.  $10 - 7x < 24$

When you multiply or divide each side of an inequality by a *negative* number, you must *reverse* the inequality symbol to maintain a true statement.

a.  $5x - 4 \geq 4x + 6$

$$x - 4 \geq 6$$

$$x \geq 10$$

b.  $10 - 7x < 24$

$$-7x < 14$$

$$x > -2$$

## EXERCISES

Solve the inequality.

22.  $-x + 2 > 7$

23.  $c - 18 < 10$

24.  $-5 + m < 21$

25.  $x - 5 < 4$

26.  $z + 6 > -2$

27.  $-3x + 4 \leq -5$

28.  $5 - 2x < -3x - 6$

29.  $-m + 3 \geq -4m + 6$

30.  $2b + 4 > -3b + 7$

31.  $13 - 6x > 10 + 4x$

32.  $4z + 8 \leq 12$

33.  $14 - 5t \geq 28$

34.  $6 - 3r < 24$

35.  $16 - 12x \leq 28$

36.  $-3x + 11 \geq 32$

## EXAMPLE 5 Absolute Value Equations and Inequalities

Solve.

a.  $|x + 8| = 4$

$$x + 8 = 4 \text{ or}$$

$$x + 8 = -4$$

$$x = -4 \text{ or } x = -12$$

b.  $|x - 5| \geq 20$

$$x - 5 \geq 20 \text{ or}$$

$$x - 5 \leq -20$$

$$x \geq 25 \text{ or } x \leq -15$$

c.  $|x + 1| < 3$

$$x + 1 < 3 \text{ and}$$

$$x + 1 > -3$$

$$x < 2 \text{ and } x > -4$$

$$-4 < x < 2$$

## EXERCISES

Solve.

37.  $|x + 5| = 12$

40.  $|1 - x| = 6$

43.  $|2x - 3| = 11$

46.  $|3x + 8| = 4$

49.  $|x - 2| \leq 8$

52.  $|6x - 4| < 8$

55.  $|11x - 11| \geq 33$

58.  $|4x - 6| > 14$

61.  $|11x + 1| > 21$

62.  $|-7x - 2| \leq -21$

64.  $|12x + 16| \leq 20$

**EXAMPLE 1****Writing and Simplifying Ratios**

a. Train A takes 35 minutes to travel its route. Train B, traveling the same route but making more stops, takes 47 minutes. What is the ratio of the time of Train A to Train B?

b. Jennie's height is 4 feet, 7 inches. Her younger sister's height is 25 inches. Find the ratio of Jennie's height to her sister's.

**SOLUTIONS**

a. 35 minutes to 47 minutes =  $\frac{35 \text{ minutes}}{47 \text{ minutes}} = \frac{35}{47}$

b. Convert 4 feet, 7 inches to inches:  $4(12) + 7 = 55$  inches  
 55 inches to 25 inches =  $\frac{55 \text{ inches}}{25 \text{ inches}} = \frac{55}{25} = \frac{11}{5}$

**EXERCISES**

Write the following ratios.

1. Basmati rice needs to cook for 20 minutes, while quinoa (another grain) cooks for 25 minutes. What is the ratio of cooking times for rice to quinoa?
2. Jonathan caught 7 fish and Geogeanne caught 4. What is the ratio of fish caught of Jonathan to Geogeanne?
3. Two sunflowers' growth was measured daily. At the end of the experiment, Sunflower A had grown from 2 inches to 2 feet, 3 inches. Sunflower B had grown from 3 inches to 2 feet, 6 inches. Find the ratio of the growth in height of Sunflower A to Sunflower B.
4. A soccer team won 22 games and lost 8. What is their win-loss ratio?
5. Charlotte's essay on pigs was 824 words in length. Wilbur's essay was only 360 words long. What is the ratio of length of Charlotte's essay to Wilbur's essay?
6. A gingham bed sheet has 220 threads per square inch while an embroidered white sheet has 180 threads per square inch. Find the ratio of threads per square inch of the gingham sheet to the white sheet.

Use the diagram at the right.

7. What is the ratio of length to width of rectangle A?

8. What is the ratio of the perimeter of rectangle A to the perimeter of rectangle B?

9. What is the ratio of the area of rectangle A to the area of rectangle B?

**EXAMPLE 2** *Distributive Property*

Solve.

a.  $4(x + 3) = 36$   
 $4x + 12 = 36$   
 $4x = 24$   
 $x = 6$

b.  $6(x + 4) + 12 = 5(x + 3) + 7$   
 $6x + 24 + 12 = 5x + 15 + 7$   
 $6x + 36 = 5x + 22$   
 $x = -14$

**EXERCISES**

Solve.

10.  $2(x + 7) = 20$

12.  $6(x - 2) = 24$

14.  $16(3 - d) = -4$

16.  $-4(x - 6) = 28$

18.  $\frac{1}{2}(10 - 9x) = \frac{3}{2}$

20.  $5(3a - 2) = 2(6a - 8)$

11.  $8(x + 6) = 24$

13.  $-10(y + 8) = -40$

15.  $7(2 - x) = 5x$

17.  $-9(5 - 3x) = 9$

19.  $\frac{2}{3}(m + 4) - 8 = \frac{11}{3}$

21.  $3(x - 1) + 3 = 4(x - 2)$

**Algebra Review**

5

# Algebra Review

5

(Six)

## EXAMPLE 3

### Solving Proportions

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Solve.

a.  $\frac{x}{8} = \frac{3}{4}$   
 $4x = 8 \cdot 3$   
 $4x = 24$   
 $x = 6$

b.  $\frac{6}{x+4} = \frac{1}{9}$   
 $6 \cdot 9 = x + 4$   
 $54 = x + 4$   
 $50 = x$

## EXERCISES

Solve.

22.  $\frac{x}{20} = \frac{1}{5}$

23.  $\frac{2}{q} = \frac{4}{18}$

24.  $\frac{7}{100} = \frac{14}{y}$

25.  $\frac{t}{27} = \frac{4}{9}$

26.  $\frac{5}{6} = \frac{4}{r}$

27.  $\frac{w}{6} = \frac{7}{17}$

28.  $\frac{27}{5} = \frac{3}{z}$

29.  $\frac{y}{50} = \frac{3}{100}$

30.  $\frac{6}{19} = \frac{m}{95}$

31.  $\frac{3}{8} = \frac{3}{2d}$

32.  $\frac{6}{5m} = \frac{6}{25}$

33.  $\frac{19}{x} = \frac{9}{5}$

34.  $\frac{3w+6}{28} = \frac{3}{4}$

35.  $\frac{6}{45} = \frac{2z+10}{15}$

36.  $\frac{3a}{11} = \frac{54}{22}$

37.  $\frac{-3}{8} = \frac{21}{2(y+1)}$

38.  $\frac{1}{18} = \frac{5}{-4(x-1)}$

39.  $\frac{3}{m+4} = \frac{9}{14}$

40.  $\frac{3}{p-6} = \frac{1}{p}$

41.  $\frac{r}{3r+1} = \frac{2}{3}$

42.  $\frac{w}{4} = \frac{9}{w}$

# Algebra Review

7

## EXAMPLE 1 Simplifying Radicals

Simplify the expression  $\sqrt{20}$ .

$$\begin{aligned}\sqrt{20} &= \sqrt{4} \cdot \sqrt{5} && \text{Use product property.} \\ &= 2\sqrt{5} && \text{Simplify.}\end{aligned}$$

### EXERCISES

Simplify the expression.

- |                  |                  |                  |
|------------------|------------------|------------------|
| 1. $\sqrt{121}$  | 2. $\sqrt{52}$   | 3. $\sqrt{45}$   |
| 4. $\sqrt{72}$   | 5. $\sqrt{40}$   | 6. $\sqrt{27}$   |
| 7. $\sqrt{80}$   | 8. $\sqrt{50}$   | 9. $\sqrt{243}$  |
| 10. $\sqrt{288}$ | 11. $\sqrt{320}$ | 12. $\sqrt{225}$ |

## EXAMPLE 2 Simplifying Radical Expressions

Simplify the radical expression.

a. $5\sqrt{3} - \sqrt{3} - \sqrt{2}$	b. $(2\sqrt{2})(5\sqrt{3})$	c. $(5\sqrt{7})^2$
$= 4\sqrt{3} - \sqrt{2}$	$= 2 \cdot 5 \cdot \sqrt{2} \cdot \sqrt{3}$	$= 5^2\sqrt{7^2}$
	$= 10\sqrt{6}$	$= 25 \cdot 7$
		$= 175$

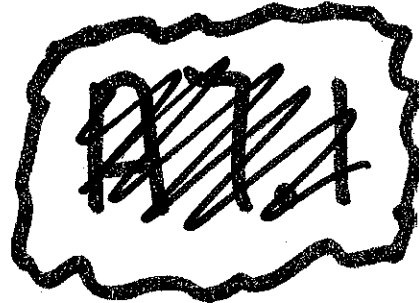
### EXERCISES

Simplify the radical expression.

- |                                |                               |  |
|--------------------------------|-------------------------------|--|
| 13. $\sqrt{75} + \sqrt{3}$     | 14. $\sqrt{50} - \sqrt{18}$   | 15. $\sqrt{64} - \sqrt{28}$            |
| 16. $\sqrt{44} + 2\sqrt{11}$   | 17. $\sqrt{125} - \sqrt{80}$  | 18. $\sqrt{242} + \sqrt{200}$          |
| 19. $-\sqrt{147} - \sqrt{243}$ | 20. $\sqrt{28} + \sqrt{63}$   | 21. $\sqrt{20} + \sqrt{45} - \sqrt{5}$ |
| 22. $(\sqrt{13})(\sqrt{26})$   | 23. $(3\sqrt{14})(\sqrt{35})$ | 24. $(\sqrt{363})(\sqrt{300})$         |
| 25. $(6\sqrt{2})(2\sqrt{2})$   | 26. $(\sqrt{18})(\sqrt{72})$  | 27. $(\sqrt{21})(\sqrt{24})$           |
| 28. $(\sqrt{32})(\sqrt{2})$    | 29. $(\sqrt{98})(\sqrt{128})$ | 30. $(5\sqrt{4})(2\sqrt{4})$           |
| 31. $(6\sqrt{5})^2$            | 32. $(4\sqrt{2})^2$           | 33. $(8\sqrt{3})^2$                    |
| 34. $(2\sqrt{3})^2$            | 35. $(5\sqrt{5})^2$           | 36. $(10\sqrt{11})^2$                  |

**EXAMPLE 3****Simplifying Quotients with Radicals**Simplify the quotient  $\frac{6}{\sqrt{5}}$ .

$$\begin{aligned}\frac{6}{\sqrt{5}} &= \frac{6}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{6\sqrt{5}}{\sqrt{5}\sqrt{5}} \\ &= \frac{6\sqrt{5}}{5}\end{aligned}$$

Multiply numerator and denominator by  $\sqrt{5}$ , to eliminate a radical in the denominator.

8

**EXERCISES**

Simplify the quotient.

37.  $\frac{4}{\sqrt{3}}$

38.  $\frac{5}{\sqrt{7}}$

39.  $\frac{2\sqrt{3}}{\sqrt{6}}$

40.  $\frac{2\sqrt{3}}{\sqrt{5}}$

41.  $\frac{\sqrt{18}}{3\sqrt{2}}$

42.  $\frac{4}{\sqrt{8}}$

43.  $\frac{16}{\sqrt{24}}$

44.  $\frac{\sqrt{5}}{\sqrt{10}}$

45.  $\frac{4}{\sqrt{12}}$

46.  $\frac{3\sqrt{5}}{\sqrt{20}}$

47.  $\frac{9}{\sqrt{52}}$

48.  $\frac{\sqrt{12}}{\sqrt{24}}$

49.  $\frac{\sqrt{18}}{\sqrt{10}}$

50.  $\frac{\sqrt{32}}{\sqrt{5}}$

51.  $\frac{\sqrt{27}}{\sqrt{45}}$

52.  $\frac{\sqrt{50}}{\sqrt{75}}$

**EXAMPLE 4****Solving Quadratic Equations**

Solve.

$x^2 - 5 = 16$

$x^2 = 21$

$x = \pm\sqrt{21}$

Add 5 to each side.

Find square roots.

**EXERCISES**

Solve.

53.  $x^2 = 9$

54.  $x^2 = 625$

55.  $x^2 = 289$

56.  $x^2 + 3 = 13$

57.  $x^2 - 4 = 12$

58.  $x^2 - 7 = 6$

59.  $7x^2 = 252$

60.  $3x^2 = 192$

61.  $6x^2 = 294$

62.  $4x^2 + 5 = 45$

63.  $2x^2 + 5 = 23$

64.  $9x^2 + 7 = 52$

65.  $11x^2 + 4 = 48$

66.  $6x^2 - 3 = 9$

67.  $10x^2 - 16 = -6$

68.  $5x^2 - 6 = 29$

69.  $8x^2 - 12 = 36$

70.  $5x^2 - 61 = 64$

71.  $x^2 + 3^2 = 5^2$

72.  $7^2 + x^2 = 25^2$

73.  $5^2 + 12^2 = x^2$



**EXAMPLE 1****Solving Literal Equations**~~18.1~~

9

Given the formula for the surface area of a right cylinder, solve for  $h$ .

$$S = 2\pi r^2 + 2\pi rh$$

$$S = 2\pi r(r + h) \quad \text{or} \quad S - 2\pi r^2 = 2\pi rh$$

$$\frac{S}{2\pi r} = r + h$$

$$\frac{(S - 2\pi r^2)}{2\pi r} = h$$

$$\frac{S}{2\pi r} - r = h$$

**EXERCISES**

Solve the literal equation for the indicated variable. Assume variables are positive.

1.  $A = \ell w; w$

2.  $V = \frac{4}{3}\pi r^3; r$

3.  $A = \frac{1}{2}bh; h$

4.  $A = \frac{1}{2}h(b_1 + b_2); b_1$

5.  $A = \pi r^2; r$

6.  $C = 2\pi r; r$

7.  $V = s^3; s$

8.  $P = 2\ell + 2w; \ell$

9.  $V = \ell wh; h$

10.  $V = \pi r^2 h; h$

11.  $S = 6s^2; s$

12.  $a^2 + b^2 = c^2; b$

**EXAMPLE 2****Algebraic Expressions**

a. Write an expression for seven less than a number.

$$x - 7$$

b. Write an equation for three less than six times a number is five times the same number plus 5, then solve.

$$6x - 3 = 5x + 5$$

$$x - 3 = 5$$

$$x = 8$$

**EXERCISES**

Write the expression or equation. Solve the equations.

13. Five plus a number

14. A number squared increased by the square root of 2

15. Twice a number decreased by fourteen

16. Six less than three times a number

17. A number plus two decreased by nine times the number

18. Half of a number plus three times the number

19. The product of five and a number decreased by seven equals thirteen.

20. Sixteen less than twice a number is 10.

**EXAMPLE 3** *Percent Problems*

- a. What number is 12% of 75?

$$x = 0.12(75)$$

$$x = 9$$

- b. 6 is what percent of 40?

$$6 = 40p$$

$$0.15 = p$$

$$p = 15\%$$

**EXERCISES**

Solve.

23. What number is 30% of 120?

24. What distance is 15% of 340 miles?

25. What number is 71% of 200?

26. How much money is 50% of \$25?

27. 34 is what percent of 136?

28. 11 dogs is what percent of 50 dogs?

29. 200 is what percent of 50?

30. 8 weeks is what percent of a year?

31. 3 is 30% of what number?

32. 16 meters is 64% of what distance?

33. If sales tax is 8%, how much tax is charged on a \$25.95 purchase?

34. 15 out of 18 players on a team came to a tournament. What percent of the players were absent?

**EXAMPLE 4** *Simplifying Rational Expressions*

Simplify.

a. 
$$\frac{8x^2 + 12x}{4x^2 + 16x}$$

$$\frac{4x(2x + 3)}{4x(x + 4)}$$

$$\frac{2x + 3}{x + 4}$$

b. 
$$\frac{y^2 - 9}{y^2 + 6y + 9}$$

$$\frac{(y + 3)(y - 3)}{(y + 3)(y + 3)}$$

$$\frac{y - 3}{y + 3}$$

**EXERCISES**

Simplify.

35. 
$$\frac{5x}{10x^2}$$

36. 
$$\frac{16a^3}{8a}$$

37. 
$$\frac{(5x^2 + x)}{(5x + 1)}$$

38. 
$$\frac{9w^3 + 27w}{3w^3 + 9w}$$

39. 
$$\frac{5a + 10}{5a - 40}$$

40. 
$$\frac{5x^2 + 15x}{30x^2 - 5x}$$

41. 
$$\frac{14d^2 - 2d}{6d^2 + 8d}$$

42. 
$$\frac{2y - 12}{24 - 2y}$$

43. 
$$\frac{36s^2 - 4s}{4s^2 - 12s}$$

44. 
$$\frac{-5h + 1}{h + 1}$$

45. 
$$\frac{t^2 - 1}{t^2 + 2t + 1}$$

46. 
$$\frac{m^2 - 4m + 4}{m^2 - 4}$$