Arithmetic Sequences (Day 1)

Definition

- **Arithmetic Sequence** - a sequence in which the difference between each term and the preceding term is always constant.

Recursive Form: $u_n = u_{n-1} + d$

$n \geq 2$

d = common difference

Explicit Form: $u_n = u_1 + (n-1)d$

$n \geq 1$

u_1 = 1st term

n = # of term

Examples of Arithmetic Sequences

Ex. 1

$+2 \quad +2 \quad +2$

5, 7, 9, 11

$u_1 = 5$, $u_2 = 7$, $d = 2$

Note:
To find "d"
$d = u_2 - u_1$

Ex. 2

$d = -1 - 4 = -5$

4, -1, -6, -11

$u_1 = 4$, $d = -5$

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Ex. Find the nth term of an arithmetic sequence with $u_1 = -3$ and a common difference of 4. Sketch a graph of the sequence. Find the 7th term.

First find the nth term.

$u_n = u_1 + (n-1)d$

$u_n = -3 + (n-1)4$

$u_n = -3 + 4n - 4$

$\boxed{u_n = 4n - 7}$

Find the 7th term.

$u_7 = 4(7) - 7$

$\boxed{u_7 = 21}$

Graph.

$\boxed{\text{Mode}} \rightarrow$ Seq

$\boxed{Y=}$ nMin = 1

$u(n) = 4n - 7$

$u(n\text{Min}) = -3$ u_1

Window:
nMin = 1
nMax = 10
xMin = 0
xMax = 10
yMin = 0
yMax = 15

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Partial Sums of an Arithmetic Sequence

- Summation Notation:

$$\sum_{k=1}^{m} C_k \quad \text{means} \quad C_1 + C_2 + C_3 + \ldots + C_m$$

Ex.

$$\sum_{n=1}^{4} 5 - 4n = (5-4(1)) + (5-4(2)) + (5-4(3)) + (5-4(4))$$
$$= 1 + -3 + -7 + -11$$
$$= -20$$

- Using a calculator to compute Sequence & Sums

Seq: List → OPS

Sum: List → Math

Sum(seq (expression, variable, begin, end))

Find the first 8 terms of $u_n = 7 - 3n$

Seq (7-3n, n, 1, 8)

$\{4, 1, -2, -5, -8, -11, -14, -17\}$

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$$\sum_{n=1}^{50} 7-3n$$

Sum(seq(7-3n, n, 1, 50)

$$\boxed{-3475}$$

Alternative Calculator Methods:

Alpha → Window → Summation

or

Math → 0: Summation

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Arithmetic Sequences (Day 2)

Partial Sums of Arithmetic Sequences cont.

Formulas

① $$\sum_{n=1}^{K} u_n = \frac{K}{2}(u_1 + u_K)$$

or

② $$\sum_{n=1}^{K} u_n = Ku_1 + \frac{K(K-1)}{2}d$$

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Ex. Use a formula to find the 14th partial sum of the arithmetic sequence

21, 15, 9, 3, . . .

$u_1 = 21 \qquad d = -6$

$u_n = 21 + (n-1)(-6)$

$u_n = 21 + 6 - 6n$

$\boxed{u_n = 27 - 6n}$

$u_{14} = 27 - 6(14)$

$= \boxed{-57}$

Find the sum with Formula 1

$\boxed{\sum_{n=1}^{14} 27 - 6n} = \frac{14}{2}(21 + (-57))$

$= -252$

Find the sum with Formula 2

$\sum_{n=1}^{14} 27 - 6n = 14(21) + \frac{14(14-1)}{2}(-6)$

$= 294 + (-546)$

$= -252$

Check with calculator:

sum(seq(27-6n, n, 1, 14))

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Ex. Find the sum of all multiples of 4 from 4 to 404.

$4, 8, 12, \ldots$ $\quad d = 4$ $\quad K = 101$

$u_1 = 4$ $\quad U_K = 404$

without formula

$u_n = 4 + (n-1)d$

$u_n = 4 + (n-1)4$

$u_n = 4n$

$$\sum_{n=1}^{101} 4n = 20{,}604$$

with formula

$K = 101$

$u_1 = 4$

$$\sum_{n=1}^{101} 4n = \frac{101}{2}(4 + 404)$$

$$= 20{,}604$$

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