Chapter 13
Counting and Probability

Section 13.1

1. union

2. intersection

3. True; the union of two sets includes those elements that are in one *or* both of the sets. The intersection consists of the elements that are in *both* sets. Thus, the intersection is a subset of the union.

4. True; every element in the universal set is either in the set A or the complement of A.

5. subset; $\subseteq$

6. finite

7. $n(A)+n(B)-n(A\cap B)$

8. True

9. $\varnothing, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}$

10. $\varnothing, \{a\}, \{b\}, \{c\}, \{d\}, \{e\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,e\}, \{b,c\}, \{b,d\}, \{b,e\}, \{c,d\}, \{c,e\}, \{d,e\}, \{a,b,c\}, \{a,b,d\}, \{a,b,e\}, \{a,c,d\}, \{a,c,e\}, \{a,d,e\}, \{b,c,d\}, \{b,c,e\}, \{b,d,e\}, \{c,d,e\}, \{a,b,c,d\}, \{a,b,c,e\}, \{a,b,d,e\}, \{a,c,d,e\}, \{b,c,d,e\}, \{a,b,c,d,e\}$

11. $n(A)=15,\ n(B)=20,\ n(A\cap B)=10$
$n(A\cup B)=n(A)+n(B)-n(A\cap B)$
$=15+20-10=25$

12. $n(A)=30,\ n(B)=40,\ n(A\cup B)=45$
$n(A\cup B)=n(A)+n(B)-n(A\cap B)$
$45=30+40-n(A\cap B)$
$n(A\cap B)=30+40-45=25$

13. $n(A\cup B)=50,\ n(A\cap B)=10,\ n(B)=20$
$n(A\cup B)=n(A)+n(B)-n(A\cap B)$
$50=n(A)+20-10$
$40=n(A)$

14. $n(A\cup B)=60,\ n(A\cap B)=40,\ n(A)=n(B)$
$n(A\cup B)=n(A)+n(B)-n(A\cap B)$
$60=n(A)+n(A)-40$
$100=2n(A)$
$n(A)=50$

15. From the figure: $n(A)=15+3+5+2=25$

16. From the figure: $n(B)=10+3+5+2=20$

17. From the figure:
$n(A \text{ or } B)=n(A\cup B)$
$=15+2+5+3+10+2=37$

18. From the figure:
$n(A \text{ and } B)=n(A\cap B)=3+5=8$

19. From the figure:
$n(A \text{ but not } C)=n(A)-n(A\cap C)=25-7=18$

20. From the figure: $n(\overline{A})=10+2+15+4=31$

21. From the figure:
$n(A \text{ and } B \text{ and } C)=n(A\cap B\cap C)=5$

22. From the figure:
$n(A \text{ or } B \text{ or } C)=n(A\cup B\cup C)$
$=15+3+5+2+10+2+15=52$

23. There are 5 choices of shirts and 3 choices of ties; there are (5)(3) = 15 different arrangements.

24. There are 5 choices of blouses and 8 choices of skirts; there are (5)(8) = 40 different outfits.

25. There are 9 choices for the first digit, and 10 choices for each of the other three digits. Thus, there are (9)(10)(10)(10) = 9000 possible four-digit numbers.

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26. There are 8 choices for the first digit, and 10 choices for each of the other four digits. Thus, there are (8)(10)(10)(10)(10) = 80,000 possible five-digit numbers.

27. Let $A = \{\text{those who will purchase a major appliance}\}$ and $B = \{\text{those who will buy a car}\}$

$n(U) = 500,\ n(A) = 200,$

$n(B) = 150,\ n(A \cap B) = 25$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 200 + 150 - 25 = 325$

$n(\text{purchase neither}) = n(U) - n(A \cup B)$

$= 500 - 325 = 175$

$n(\text{purchase only a car}) = n(B) - n(A \cup B)$

$= 150 - 25 = 125$

28. Let $A = \{\text{those who will attend Summer Session I}\}$ and $B = \{\text{those who will attend Summer Session II}\}$

$n(A) = 200,\ n(B) = 150,\ n(A \cap B) = 75,$

$n(\overline{A \cup B}) = 275$

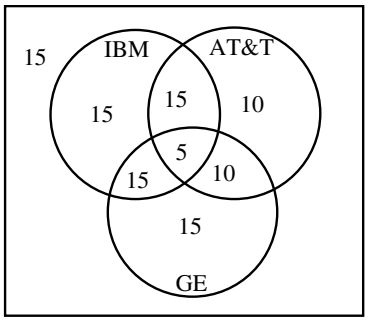
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 200 + 150 - 75 = 275$

$n(U) = n(A \cup B) + n(\overline{A \cup B}) = 275 + 275 = 550$

550 students participated in the survey.

29. Construct a Venn diagram:

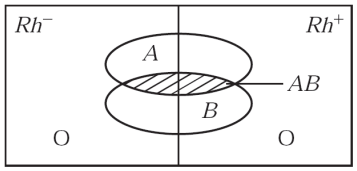


(a) 15 (b) 15

(c) 15 (d) 25

(e) 40

30. Construct a Venn diagram:



There are 8 different kinds of blood: A-Rh+, B-Rh+, AB-Rh+, O-Rh+, A-Rh–, B-Rh–, AB-Rh–, O-Rh–

31. **a.** $n(\text{widowed or divorced}) = n(\text{widowed}) + n(\text{divorced})$

$= 3.1 + 10.7$

$= 13.8$

There were 13.8 million males 18 years old and older who were widowed or divorced.

b. $n(\text{married, widowed or divorced})$

$= n(\text{married}) + n(\text{widowed}) + n(\text{divorced})$

$= 65.7 + 3.1 + 10.7 = 79.5$

There were 79.5 million males 18 years old and older who were married, widowed, or divorced.

32. **a.** $n(\text{widowed or divorced}) = n(\text{widowed}) + n(\text{divorced})$

$= 11.2 + 14.6$

$= 25.8$

There were 25.8 million females 18 years old and older who were widowed or divorced.

b. $n(\text{married, widowed or divorced})$

$= n(\text{married}) + n(\text{widowed}) + n(\text{divorced})$

$= 66.7 + 11.2 + 14.6$

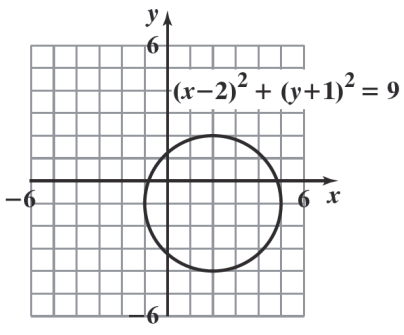
$= 92.5$

There were 92.5 million females 18 years old and older who were married, widowed, or divorced.

33. There are 8 choices for the DOW stocks, 15 choices for the NASDAQ stocks, and 4 choices for the global stocks. Thus, there are (8)(15)(4) = 480 different portfolios.

34 – 35. Answers will vary.

36. The graph is a circle with center at $(2, -1)$ and radius 3.



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37. $a = 2,\ b = 2\ \ c = 3$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{2^2 + 3^2 - 2^2}{2(2)(3)} = \frac{3}{4}$$

$$A = \cos^{-1}\left(\frac{3}{4}\right) \approx 41.4^\circ$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{2^2 + 3^2 - 32^2}{2(2)(3)} = \frac{3}{4}$$

$$B = \cos^{-1}\left(\frac{3}{4}\right) \approx 41.4^\circ$$

$$C = 180^\circ - A - B \approx 180^\circ - 41.4^\circ - 41.4^\circ = 97.2^\circ$$

38. $f(x) = (x-2)(x^2 - 3x - 10)$

$$0 = (x-2)(x-5)(x+2)$$

$(x-2) = 0 \quad (x-5) = 0 \quad (x+2) = 0$

$x = 2 \qquad x = 5 \qquad x = -2$

The zeros are $2, 5, -2$.

39. $\log_3 x + \log_3 2 = -2$

$$\log_3(2x) = -2$$

$$3^{-2} = 2x$$

$$\frac{1}{9} = 2x$$

$$x = \frac{1}{18}$$

Section 13.2

1. 1; 1

2. False; $n! = \dfrac{(n+1)!}{n+1}$

3. permutation

4. combination

5. $P(n, r) = \dfrac{n!}{(n-r)!}$

6. $C(n, r) = \dfrac{n!}{(n-r)!\,r!}$

7. $P(6, 2) = \dfrac{6!}{(6-2)!} = \dfrac{6!}{4!} = \dfrac{6\cdot 5\cdot 4!}{4!} = 30$

8. $P(7, 2) = \dfrac{7!}{(7-2)!} = \dfrac{7!}{5!} = \dfrac{7\cdot 6\cdot 5!}{5!} = 42$

9. $P(4, 4) = \dfrac{4!}{(4-4)!} = \dfrac{4!}{0!} = \dfrac{4\cdot 3\cdot 2\cdot 1}{1} = 24$

10. $P(8, 8) = \dfrac{8!}{(8-8)!} = \dfrac{8!}{0!}$

$$= \frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{1} = 40,320$$

11. $P(7, 0) = \dfrac{7!}{(7-0)!} = \dfrac{7!}{7!} = 1$

12. $P(9, 0) = \dfrac{9!}{(9-0)!} = \dfrac{9!}{9!} = 1$

13. $P(8, 4) = \dfrac{8!}{(8-4)!} = \dfrac{8!}{4!} = \dfrac{8\cdot 7\cdot 6\cdot 5\cdot 4!}{4!} = 1680$

14. $P(8, 3) = \dfrac{8!}{(8-3)!} = \dfrac{8!}{5!} = \dfrac{8\cdot 7\cdot 6\cdot 5!}{5!} = 336$

15. $C(8, 2) = \dfrac{8!}{(8-2)!\,2!} = \dfrac{8!}{6!\,2!} = \dfrac{8\cdot 7\cdot 6!}{6!\cdot 2\cdot 1} = 28$

16. $C(8, 6) = \dfrac{8!}{(8-6)!\,6!} = \dfrac{8!}{2!\,6!} = \dfrac{8\cdot 7\cdot 6!}{6!\cdot 2\cdot 1} = 28$

17. $C(7, 4) = \dfrac{7!}{(7-4)!\,4!} = \dfrac{7!}{3!\,4!} = \dfrac{7\cdot 6\cdot 5\cdot 4!}{4!\cdot 3\cdot 2\cdot 1} = 35$

18. $C(6, 2) = \dfrac{6!}{(6-2)!\,2!} = \dfrac{6!}{4!\,2!} = \dfrac{6\cdot 5\cdot 4!}{4!\cdot 2\cdot 1} = 15$

19. $C(15, 15) = \dfrac{15!}{(15-15)!\,15!} = \dfrac{15!}{0!\,15!} = \dfrac{15!}{15!\cdot 1} = 1$

20. $C(18, 1) = \dfrac{18!}{(18-1)!\,1!} = \dfrac{18!}{17!\,1!} = \dfrac{18\cdot 17!}{17!\cdot 1} = 18$

21. $C(26, 13) = \dfrac{26!}{(26-13)!\,13!} = \dfrac{26!}{13!\,13!} = 10,400,600$

22. $C(18, 9) = \dfrac{18!}{(18-9)!\,9!} = \dfrac{18!}{9!\,9!} = 48,620$

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23. {*abc*, *abd*, *abe*, *acb*, *acd*, *ace*, *adb*, *adc*, *ade*, *aeb*, *aec*, *aed*, *bac*, *bad*, *bae*, *bca*, *bcd*, *bce*, *bda*, *bdc*, *bde*, *bea*, *bec*, *bed*, *cab*, *cad*, *cae*, *cba*, *cbd*, *cbe*, *cda*, *cdb*, *cde*, *cea*, *ceb*, *ced*, *dab*, *dac*, *dae*, *dba*, *dbc*, *dbe*, *dca*, *dcb*, *dce*, *dea*, *deb*, *dec*, *eab*, *eac*, *ead*, *eba*, *ebc*, *ebd*, *eca*, *ecb*, *ecd*, *eda*, *edb*, *edc*}

$$P(5,3)=\frac{5!}{(5-3)!}=\frac{5!}{2!}=\frac{5\cdot4\cdot3\cdot2!}{2!}=60$$

24. {*ab*, *ac*, *ad*, *ae*, *ba*, *bc*, *bd*, *be*, *ca*, *cb*, *cd*, *ce*, *da*, *db*, *dc*, *de*, *ea*, *eb*, *ec*, *ed*}

$$P(5,2)=\frac{5!}{(5-2)!}=\frac{5!}{3!}=\frac{5\cdot4\cdot3!}{3!}=20$$

25. {123, 124, 132, 134, 142, 143, 213, 214, 231, 234, 241, 243, 312, 314, 321, 324, 341, 342, 412, 413, 421, 423, 431, 432}

$$P(4,3)=\frac{4!}{(4-3)!}=\frac{4!}{1!}=\frac{4\cdot3\cdot2\cdot1}{1}=24$$

26. {123, 124, 125, 126, 132, 134, 135, 136, 142, 143, 145, 146, 152, 153, 154, 156, 162, 163, 164, 165, 213, 214, 215, 216, 231, 234, 235, 236, 241, 243, 245, 246, 251, 253, 254, 256, 261, 263, 264, 265, 312, 314, 315, 316, 321, 324, 325, 326, 341, 342, 345, 346, 351, 352, 354, 356, 361, 362, 364, 365, 412, 413, 415, 416, 421, 423, 425, 426, 431, 432, 435, 436, 451, 452, 453, 456, 461, 462, 463, 465, 512, 513, 514, 516, 521, 523, 524, 526, 531, 532, 534, 536, 541, 542, 543, 546, 561, 562, 563, 564, 612, 613, 614, 615, 621, 623, 624, 625, 631, 632, 634, 635, 641, 642, 643, 645, 651, 652, 653, 654}

$$P(6,3)=\frac{6!}{(6-3)!}=\frac{6!}{3!}=\frac{6\cdot5\cdot4\cdot3!}{3!}=120$$

27. {*abc*, *abd*, *abe*, *acd*, *ace*, *ade*, *bcd*, *bce*, *bde*, *cde*}

$$C(5,3)=\frac{5!}{(5-3)!\,3!}=\frac{5\cdot4\cdot3!}{2\cdot1\cdot3!}=10$$

28. {*ab*, *ac*, *ad*, *ae*, *bc*, *bd*, *be*, *cd*, *ce*, *de*}

$$C(5,2)=\frac{5!}{(5-2)!\,2!}=\frac{5\cdot4\cdot3!}{3!\cdot2\cdot1}=10$$

29. {123, 124, 134, 234}

$$C(4,3)=\frac{4!}{(4-3)!\,3!}=\frac{4\cdot3!}{1!\,3!}=4$$

30. {123, 124, 125, 126, 134, 135, 136, 145, 146, 156, 234, 235, 236, 245, 246, 256, 345, 346, 356, 456}

$$C(6,3)=\frac{6!}{(6-3)!\,3!}=\frac{6\cdot5\cdot4\cdot3!}{3\cdot2\cdot1\cdot3!}=20$$

31. There are 4 choices for the first letter in the code and 4 choices for the second letter in the code; there are (4)(4) = 16 possible two-letter codes.

32. There are 5 choices for the first letter in the code and 5 choices for the second letter in the code; there are (5)(5) = 25 possible two-letter codes.

33. There are two choices for each of three positions; there are (2)(2)(2) = 8 possible three-digit numbers.

34. There are ten choices for each of three positions; there are (10)(10)(10) = 1000 possible three-digit numbers. (Note this is if we allow numbers with initial zeros such as 012.)

35. To line up the four people, there are 4 choices for the first position, 3 choices for the second position, 2 choices for the third position, and 1 choice for the fourth position. Thus there are (4)(3)(2)(1) = 24 possible ways four people can be lined up.

36. To stack the five boxes, there are 5 choices for the first position, 4 choices for the second position, 3 choices for the third position, 2 choices for the fourth position, and 1 choice for the fifth position. Thus, there are (5)(4)(3)(2)(1) = 120 possible ways five boxes can be stacked.

37. Since no letter can be repeated, there are 5 choices for the first letter, 4 choices for the second letter, and 3 choices for the third letter. Thus, there are (5)(4)(3) = 60 possible three-letter codes.

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38. Since no letter can be repeated, there are 6 choices for the first letter, 5 choices for the second letter, 4 choices for the third letter, and 3 choices for the fourth letter. Thus, there are (6)(5)(4)(3) = 360 possible three-letter codes.

39. There are 26 possible one-letter names. There are (26)(26) = 676 possible two-letter names. There are (26)(26)(26) = 17,576 possible three-letter names. Thus, there are 26 + 676 + 17,576 = 18,278 possible companies that can be listed on the New York Stock Exchange.

40. There are (26)(26)(26)(26) = 456,976 possible four-letter names. There are 26)(26)(26)(26)(26) = 11,881,376 possible five-letter names. Thus, there are 456,976 + 11,881,376 = 12,338,352 possible companies that can be listed on the NASDAQ.

41. A committee of 4 from a total of 7 students is given by:

$$C(7,4) = \frac{7!}{(7-4)!\,4!} = \frac{7!}{3!\,4!} = \frac{7\cdot 6\cdot 5\cdot 4!}{3\cdot 2\cdot 1\cdot 4!} = 35$$

35 committees are possible.

42. A committee of 3 from a total of 8 professors is given by:

$$C(8,3) = \frac{8!}{(8-3)!\,3!} = \frac{8!}{5!\,3!} = \frac{8\cdot 7\cdot 6\cdot 5!}{3\cdot 2\cdot 1\cdot 5!} = 56$$

56 committees are possible.

43. There are 2 possible answers for each question. Therefore, there are $2^{10} = 1024$ different possible arrangements of the answers.

44. There are 4 possible answers for each question. Therefore, there are $4^5 = 1024$ different possible arrangements of the answers.

45. There are 5 choices for the first position, 4 choices for the second position, 3 choices for the third position, 2 choices for the fourth position, and 1 choice for the fifth position. Thus, there are (5)(4)(3)(2)(1) = 120 possible arrangements of the books.

46. a. There are 26 choices for each of the first two positions, and 10 choices for each of the next four positions. Thus, there are (26)(26)(10)(10)(10)(10) = 6,760,000 possible license plates.

b. There are 26 choices for each of the first two positions, 10 choices for the first digit, 9 choices for the second digit, 8 choices for the third digit, and 7 choices for the fourth digit. Thus, there are (26)(26)(10)(9)(8)(7) = 3,407,040 possible license plates.

c. There are 26 choices for the first letter, 25 choices for the second letter, 10 choices for the first digit, 9 choices for the second digit, 8 choices for the third digit, and 7 choices for the fourth digit. Thus, there are (26)(25)(10)(9)(8)(7) = 3,276,000 possible license plates.

47. The 1st person can have any of 365 days, the 2nd person can have any of the remaining 364 days. Thus, there are (365)(364) = 132,860 possible ways two people can have different birthdays.

48. The first person can have any of 365 days, the second person can have any of the remaining 364 days, the third person can have any of the remaining 363 days, the fourth person can have any of the remaining 362 days, and the fifth person can have any of the remaining 361 days. Thus, there are (365)(364)(363)(362)(361) = 6,302,555,018,760 possible ways five people can have different birthdays.

49. Choosing 2 boys from the 4 boys can be done $C(4,2)$ ways. Choosing 3 girls from the 8 girls can be done in $C(8,3)$ ways. Thus, there are a total of:

$$\begin{aligned} C(4,2)\cdot C(8,3) &= \frac{4!}{(4-2)!\,2!}\cdot\frac{8!}{(8-3)!\,3!} \\ &= \frac{4!}{2!\,2!}\cdot\frac{8!}{5!\,3!} \\ &= \frac{4\cdot 3\cdot 2!}{2\cdot 1\cdot 2!}\cdot\frac{8\cdot 7\cdot 6\cdot 5!}{5!\,3!} \\ &= 6\cdot 56 \\ &= 336 \end{aligned}$$

50. The committee is made up of 2 of 4 administrators, 3 of 8 faculty members, and 5 of 20 students. The number of possible committees is:

$$\begin{aligned} & C(4,2)\cdot C(8,3)\cdot C(20,5) \\ &= \frac{4!}{(4-2)!\,2!}\cdot\frac{8!}{(8-3)!\,3!}\cdot\frac{20!}{(20-5)!\,5!} \\ &= \frac{4!}{2!\,2!}\cdot\frac{8!}{5!\,3!}\cdot\frac{20!}{15!\,5!} \\ &= \frac{4!}{2\cdot 1\cdot 2\cdot 1}\cdot\frac{8\cdot 7\cdot 6\cdot 5!}{5\cdot 4!\,3!}\cdot\frac{20\cdot 19\cdot 18\cdot 17\cdot 16\cdot 15!}{15!\,5!} \\ &= 5{,}209{,}344 \text{ possible committees} \end{aligned}$$

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51. This is a permutation with repetition. There are $\frac{9!}{2!\,2!} = 90,720$ different words.

52. This is a permutation with repetition. There are $\frac{11!}{2!\,2!\,2!} = 4,989,600$ different words.

53. **a.** $C(7,2)\cdot C(3,1) = 21\cdot 3 = 63$

b. $C(7,3)\cdot C(3,0) = 35\cdot 1 = 35$

c. $C(3,3)\cdot C(7,0) = 1\cdot 1 = 1$

54. **a.** $C(15,5)\cdot C(10,0) = 3003\cdot 1 = 3003$

b. $C(15,3)\cdot C(10,2) = 455\cdot 45 = 20,475$

c. $C(15,4)\cdot C(10,1) + C(15,5)\cdot C(10,0)$
$= 1365\cdot 10 + 3003\cdot 1$
$= 13,650 + 3003$
$= 16,653$

55. There are $C(100, 22)$ ways to form the first committee. There are 78 senators left, so there are $C(78, 13)$ ways to form the second committee. There are $C(65, 10)$ ways to form the third committee. There are $C(55, 5)$ ways to form the fourth committee. There are $C(50, 16)$ ways to form the fifth committee. There are C(34, 17) ways to form the sixth committee. There are $C(17, 17)$ ways to form the seventh committee.
The total number of committees
$= C(100,22)\cdot C(78,13)\cdot C(65,10)\cdot C(55,5)$
$\cdot C(50,16)\cdot C(34,17)\cdot C(17,17)$
$\approx 1.157\times 10^{76}$

56. The team is made up of 5 of 10 linemen, 3 of 10 linebackers, and 3 of 5 safeties. The number of possible teams is:
$C(10,5)\cdot C(10,3)\cdot C(5,3) = 252\cdot 120\cdot 10$
$= 302,400$
There are 302,400 possible defensive teams.

57. There are 9 choices for the first position, 8 choices for the second position, 7 for the third position, etc. There are $9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1 = 9! = 362,880$ possible batting orders.

58. There are 8 choices for the first position, 7 choices for the second position, 6 for the third position, etc. and 1 choice for the last position. There are $8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 1 = 8!\cdot 1 = 40,320$ possible batting orders.

59. The team must have 1 pitcher and 8 position players (non-pitchers). For pitcher, choose 1 player from a group of 4 players, i.e., $C(4, 1)$. For position players, choose 8 players from a group of 11 players, i.e., $C(11, 8)$. Thus, the number different teams possible is $C(4,1)\cdot C(11,8) = 4\cdot 165 = 660$.

60. Consider the ways that the American League can win. Then multiply by 2 to get the total for both leagues. To win the World Series, the last game must be won. There is 1 way to win in four games. To win in 5 games, three of the first four must be won, so there are $C(4, 3) = 4$ ways to win in 5 games. To win in 6 games, three of the first five must be won, so there are $C(5, 3) = 10$ ways to win in 6 games. To win in 7 games, three of the first six must be won, so there are $C(6, 3) = 20$ ways to win in 7 games. Therefore, there are $1 + 4 + 10 + 20 = 35$ ways the American League can win the World Series. There are also 35 ways the National League can win the World Series. There are a total of 70 different sequences possible.

61. Choose 2 players from a group of 6 players. Thus, there are $C(6,2) = 15$ different teams possible.

62. Choose 1 of 2 centers, 2 of 3 guards, and 2 of 7 forwards. There are $C(2,1)\cdot C(3,2)\cdot C(7,2) = 2\cdot 3\cdot 21 = 126$ different teams possible.

63. **a.** If numbers can be repeated, there are (50)(50)(50) = 125,000 different lock combinations. If no number can be repeated, then there are $50\cdot 49\cdot 48 = 117,600$ different lock combinations.

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b. Answers will vary. Typical combination locks require two full clockwise rotations to the first number, followed by a full counter-clockwise rotation past the first number to the second number, followed by a clockwise rotation to the third number (not past the second). This is not clear from the given directions. Perhaps a better name for a *combination* lock would be a *permutation* lock since the order in which the numbers are entered matters.

64 – 65. Answers will vary.

66. A permutation is an ordered arrangement of objects while with a combination order does not matter. For example, the number of ways the 11 teams in the Big Ten can come in first, second, and third would be a permutation problem. The number of ways to pick 6 numbers in the Illinois State lottery is a combination problem because the order in which the numbers are selected is irrelevant.

67. First solve for θ

$$s = r\theta$$
$$5 = 4\theta$$
$$\theta = \frac{5}{4}$$
$$A = \frac{1}{2}r^2\theta$$
$$= \frac{1}{2}(4)^2\left(\frac{5}{4}\right)$$
$$= 10 \text{ ft}^2$$

68. $(g \circ f)(x) = g(2x-1)$

$$= (2x-1)^2 + (2x-1) - 2$$
$$= 4x^2 - 4x + 1 + 2x - 1 - 2$$
$$= 4x^2 - 2x - 2$$

69. $\sin 75° = \sin(45° + 30°)$

$$= \sin 45°\cos 30° + \cos 45°\sin 30°$$
$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$
$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$\cos 15° = \cos(45° - 30°)$

$$= \cos 45°\cos 30° + \sin 45°\sin 30°$$
$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$
$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

Also, $\cos 15° = \sqrt{\frac{1+\cos 30°}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$

$$= \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

70. $a_n = a_1 r^{n-1}$

$$= 5(-2)^{5-1} = 5(-2)^4$$
$$= 80$$

Section 13.3

1. equally likely

2. complement

3. False; probability may equal 0. In such cases, the corresponding event will never happen.

4. True; in a valid probability model, all probabilities are between 0 and 1, and the sum of the probabilities is 1.

5. Probabilities must be between 0 and 1, inclusive. Thus, 0, 0.01, 0.35, and 1 could be probabilities.

6. Probabilities must be between 0 and 1, inclusive. Thus, $\frac{1}{2}, \frac{3}{4}, \frac{2}{3}$, and 0 could be probabilities.

7. All the probabilities are between 0 and 1.
The sum of the probabilities is
0.2 + 0.3 + 0.1 + 0.4 = 1.
This is a probability model.

8. All the probabilities are between 0 and 1.
The sum of the probabilities is
0.4 + 0.3 + 0.1 + 0.2 = 1.
This is a probability model.

9. All the probabilities are between 0 and 1.
The sum of the probabilities is
0.3 + 0.2 + 0.1 + 0.3 = 0.9.
This is not a probability model.

10. One probability is not between 0 and 1.
This is not a probability model.

11. The sample space is: $S = \{HH, HT, TH, TT\}$.
Each outcome is equally likely to occur; so

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$P(E) = \frac{n(E)}{n(S)}$. The probabilities are:

$P(HH) = \frac{1}{4},\ P(HT) = \frac{1}{4},\ P(TH) = \frac{1}{4},\ P(TT) = \frac{1}{4}$.

12. The sample space is: $S = \{HH, HT, TH, TT\}$. Each outcome is equally likely to occur; so $P(E) = \frac{n(E)}{n(S)}$. The probabilities are:

$P(HH) = \frac{1}{4},\ P(HT) = \frac{1}{4},\ P(TH) = \frac{1}{4},\ P(TT) = \frac{1}{4}$

13. The sample space of tossing two fair coins and a fair die is:

$S = \{HH1, HH2, HH3, HH4, HH5, HH6,$
$HT1, HT2, HT3, HT4, HT5, HT6, TH1,$
$TH2, TH3, TH4, TH5, TH6, TT1, TT2,$
$TT3, TT4, TT5, TT6\}$

There are 24 equally likely outcomes and the probability of each is $\frac{1}{24}$.

14. The sample space of tossing a fair coin, a fair die, and a fair coin is:

$S = \{H1H, H2H, H3H, H4H, H5H, H6H,$
$H1T, H2T, H3T, H4T, H5T, H6T,$
$T1H, T2H, T3H, T4H, T5H, T6H,$
$T1T, T2T, T3T, T4T, T5T, T6T\}$

There are 24 equally likely outcomes and the probability of each is $\frac{1}{24}$.

15. The sample space for tossing three fair coins is:

$S = \{HHH, HHT, HTH, HTT, THH, THT,$
$TTH, TTT\}$

There are 8 equally likely outcomes and the probability of each is $\frac{1}{8}$.

16. The sample space for tossing one fair coin three times is: $S = \{HHH, HHT, HTH, HTT, THH,$ $THT, TTH, TTT\}$. There are 8 equally likely outcomes and the probability of each is $\frac{1}{8}$.

17. The sample space is: S = {1 Yellow, 1 Red, 1 Green, 2 Yellow, 2 Red, 2 Green, 3 Yellow, 3 Red, 3 Green, 4 Yellow, 4 Red, 4 Green}
There are 12 equally likely events and the probability of each is $\frac{1}{12}$. The probability of getting a 2 or 4 followed by a Red is

$P(2 \text{ Red}) + P(4 \text{ Red}) = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$.

18. The sample space is: S = {Forward Yellow, Forward Red, Forward Green, Backward Yellow, Backward Red, Backward Green}
There are 6 equally likely events and the probability of each is $\frac{1}{6}$. The probability of getting Forward followed by Yellow or Green is:

$P(\text{Forward Yellow}) + P(\text{Forward Green}) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

19. The sample space is:
S = {1 Yellow Forward, 1 Yellow Backward, 1 Red Forward, 1 Red Backward, 1 Green Forward, 1 Green Backward, 2 Yellow Forward, 2 Yellow Backward, 2 Red Forward, 2 Red Backward, 2 Green Forward, 2 Green Backward, 3 Yellow Forward, 3 Yellow Backward, 3 Red Forward, 3 Red Backward, 3 Green Forward, 3 Green Backward, 4 Yellow Forward, 4 Yellow Backward, 4 Red Forward, 4 Red Backward, 4 Green Forward, 4 Green Backward}
There are 24 equally likely events and the probability of each is $\frac{1}{24}$. The probability of getting a 1, followed by a Red or Green, followed by a Backward is:

$P(1 \text{ Red Backward}) + P(1 \text{ Green Backward})$
$= \frac{1}{24} + \frac{1}{24} = \frac{1}{12}$

20. The sample space is:
S = {Yellow 1 Forward, Yellow 1 Backward, Red 1 Forward, Red 1 Backward, Green 1 Forward, Green 1 Backward, Yellow 2 Forward, Yellow 2 Backward, Red 2 Forward, Red 2 Backward, Green 2 Forward, Green 2 Backward, Yellow 3 Forward, Yellow 3 Backward, Red 3 Forward, Red 3 Backward, Green 3 Forward, Green 3 Backward, Yellow 4 Forward, Yellow 4 Backward, Red 4 Forward, Red 4 Backward, Green 4 Forward, Green 4 Backward}
There are 24 equally likely events and the probability of each is $\frac{1}{24}$.

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The probability of getting a Yellow, followed by a 2 or 4, followed by a Forward is

$P(\text{Yellow 2 Forward}) + P(\text{Yellow 4 Forward})$

$= \frac{1}{24} + \frac{1}{24} = \frac{1}{12}$

21. The sample space is:

S = {1 1 Yellow, 1 1 Red, 1 1 Green, 1 2 Yellow, 1 2 Red, 1 2 Green, 1 3 Yellow, 1 3 Red, 1 3 Green, 1 4 Yellow, 1 4 Red, 1 4 Green, 2 1 Yellow, 2 1 Red, 2 1 Green, 2 2 Yellow, 2 2 Red, 2 2 Green, 2 3 Yellow, 2 3 Red, 2 3 Green, 2 4 Yellow, 2 4 Red, 2 4 Green, 3 1 Yellow, 3 1 Red, 3 1 Green, 3 2 Yellow, 3 2 Red, 3 2 Green, 3 3 Yellow, 3 3 Red, 3 3 Green, 3 4 Yellow, 3 4 Red, 3 4 Green, 4 1 Yellow, 4 1 Red, 4 1 Green, 4 2 Yellow, 4 2 Red, 4 2 Green, 4 3 Yellow, 4 3 Red, 4 3 Green, 4 4 Yellow, 4 4 Red, 4 4 Green}

There are 48 equally likely events and the probability of each is $\frac{1}{48}$. The probability of getting a 2, followed by a 2 or 4, followed by a Red or Green is

$P(\text{2 2 Red}) + P(\text{2 4 Red}) + P(\text{2 2 Green}) + P(\text{2 4 Green})$

$= \frac{1}{48} + \frac{1}{48} + \frac{1}{48} + \frac{1}{48} = \frac{1}{12}$

22. The sample space is:

S = {Forward 11, Forward 12, Forward 13, Forward 14, Forward 21, Forward 22, Forward 23, Forward 24, Forward 31, Forward 32, Forward 33, Forward 34, Forward 41, Forward 42, Forward 43, Forward 44, Backward 11, Backward 12, Backward 13, Backward 14, Backward 21, Backward 22, Backward 23, Backward 24, Backward 31, Backward 32, Backward 33, Backward 34, Backward 41, Backward 42, Backward 43, Backward 44}

There are 32 equally likely events and the probability of each is $\frac{1}{32}$. The probability of getting a Forward, followed by a 1 or 3, followed by a 2 or 4 is

$P(\text{Fwd 12}) + P(\text{Fwd 14}) + P(\text{Fwd 32}) + P(\text{Fwd 34})$

$= \frac{1}{32} + \frac{1}{32} + \frac{1}{32} + \frac{1}{32} = \frac{1}{8}$

23. A, B, C, F

24. A (equally likely outcomes)

25. B

26. F

27. Let $P(\text{tails}) = x$, then $P(\text{heads}) = 4x$

$$x + 4x = 1$$
$$5x = 1$$
$$x = \frac{1}{5}$$

$P(\text{tails}) = \frac{1}{5}, \quad P(\text{heads}) = \frac{4}{5}$

28. Let $P(\text{heads}) = x$, then $P(\text{tails}) = 2x$

$$x + 2x = 1$$
$$3x = 1$$
$$x = \frac{1}{3}$$

$P(\text{heads}) = \frac{1}{3}, \quad P(\text{tails}) = \frac{2}{3}$

29. $P(2) = P(4) = P(6) = x$

$P(1) = P(3) = P(5) = 2x$

$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$

$$2x + x + 2x + x + 2x + x = 1$$
$$9x = 1$$
$$x = \frac{1}{9}$$

$P(2) = P(4) = P(6) = \frac{1}{9}$

$P(1) = P(3) = P(5) = \frac{2}{9}$

30. $P(1) = P(2) = P(3) = P(4) = P(5) = x; \quad P(6) = 0$

$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$

$$x + x + x + x + x + 0 = 1$$
$$5x = 1$$
$$x = \frac{1}{5}$$

$P(1) = P(2) = P(3) = P(4) = P(5) = \frac{1}{5}; \quad P(6) = 0$

31. $P(E) = \frac{n(E)}{n(S)} = \frac{n\{1,2,3\}}{10} = \frac{3}{10}$

32. $P(F) = \frac{n(F)}{n(S)} = \frac{n\{3,5,9,10\}}{10} = \frac{4}{10} = \frac{2}{5}$

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33. $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{n\{2,4,6,8,10\}}{10} = \dfrac{5}{10} = \dfrac{1}{2}$

34. $P(F) = \dfrac{n(F)}{n(S)} = \dfrac{n\{1,3,5,7,9\}}{10} = \dfrac{5}{10} = \dfrac{1}{2}$

35. $P(\text{white}) = \dfrac{n(\text{white})}{n(S)} = \dfrac{5}{5+10+8+7} = \dfrac{5}{30} = \dfrac{1}{6}$

36. $P(\text{black}) = \dfrac{n(\text{black})}{n(S)} = \dfrac{7}{5+10+8+7} = \dfrac{7}{30}$

37. The sample space is: S = {BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG}

$$P(3 \text{ boys}) = \frac{n(3 \text{ boys})}{n(S)} = \frac{1}{8}$$

38. The sample space is: S = {BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG}

$$P(3 \text{ girls}) = \frac{n(3 \text{ girls})}{n(S)} = \frac{1}{8}$$

39. The sample space is: S = {BBBB, BBBG, BBGB, BGBB, GBBB, BBGG, BGBG, GBBG, BGGB, GBGB, GGBB, BGGG, GBGG, GGBG, GGGB, GGGG}

$$P(1 \text{ girl, } 3 \text{ boys}) = \frac{n(1 \text{ girl, } 3 \text{ boys})}{n(S)} = \frac{4}{16} = \frac{1}{4}$$

40. The sample space is: S = {BBBB, BBBG, BBGB, BGBB, GBBB, BBGG, BGBG, GBBG, BGGB, GBGB, GGBB, BGGG, GBGG, GGBG, GGGB, GGGG}

$$P(2 \text{ girls, } 2 \text{ boys}) = \frac{n(2 \text{ girls, } 2 \text{ boys})}{n(S)} = \frac{6}{16} = \frac{3}{8}$$

41. $P(\text{sum of two dice is 7}) = \dfrac{n(\text{sum of two dice is 7})}{n(S)}$

$$= \frac{n\{1,6 \text{ or } 2,5 \text{ or } 3,4 \text{ or } 4,3 \text{ or } 5,2 \text{ or } 6,1\}}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

42. $P(\text{sum of two dice is 11}) = \dfrac{n(\text{sum of two dice is 11})}{n(S)}$

$$= \frac{n\{5,6 \text{ or } 6,5\}}{n(S)} = \frac{2}{36} = \frac{1}{18}$$

43. $P(\text{sum of two dice is 3}) = \dfrac{n(\text{sum of two dice is 3})}{n(S)}$

$$= \frac{n\{1,2 \text{ or } 2,1\}}{n(S)} = \frac{2}{36} = \frac{1}{18}$$

44. $P(\text{sum of two dice is 12}) = \dfrac{n(\text{sum of two dice is 12})}{n(S)}$

$$= \frac{n\{6,6\}}{n(S)} = \frac{1}{36}$$

45. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= 0.25 + 0.45 - 0.15 = 0.55$$

46. $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$$= 0.25 + 0.45 - 0.6 = 0.10$$

47. $P(A \cup B) = P(A) + P(B) = 0.25 + 0.45 = 0.70$

48. $P(A \cap B) = 0$

49. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.85 = 0.60 + P(B) - 0.05$

$P(B) = 0.85 - 0.60 + 0.05 = 0.30$

50. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.65 = P(A) + 0.30 - 0.15$

$P(A) = 0.65 - 0.30 + 0.15 = 0.50$

51. $P(\text{theft not cleared}) = 1 - P(\text{ theft cleared})$

$$= 1 - 0.142$$

$$= 0.858$$

52. $P(\text{does not own a pet}) = 1 - P(\text{owns a pet})$

$$= 1 - 0.65$$

$$= 0.35$$

53. $P(\text{does not own dog}) = 1 - P(\text{owns dog})$

$$= 1 - 0.44$$

$$= 0.56$$

54. $P(\text{not in engineering}) = 1 - P(\text{in engineering})$

$$= 1 - 0.17$$

$$= 0.83$$

55. $P(\text{never gambled online}) = 1 - P(\text{gambled online})$

$$= 1 - 0.064$$

$$= 0.936$$

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56. $P(\text{not samoas/caramel deLites})$
$= 1 - P(\text{samoas/caramel deLites})$
$= 1 - 0.19$
$= 0.81$

57. $P(\text{white or green}) = P(\text{white}) + P(\text{green})$
$$= \frac{n(\text{white}) + n(\text{green})}{n(S)}$$
$$= \frac{9+8}{9+8+3} = \frac{17}{20}$$

58. $P(\text{white or orange}) = P(\text{white}) + P(\text{orange})$
$$= \frac{n(\text{white}) + n(\text{orange})}{n(S)}$$
$$= \frac{9+3}{9+8+3} = \frac{12}{20} = \frac{3}{5}$$

59. $P(\text{not white}) = 1 - P(\text{white})$
$$= 1 - \frac{n(\text{white})}{n(S)}$$
$$= 1 - \frac{9}{20} = \frac{11}{20}$$

60. $P(\text{not green}) = 1 - P(\text{green})$
$$= 1 - \frac{n(\text{green})}{n(S)}$$
$$= 1 - \frac{8}{20} = \frac{12}{20} = \frac{3}{5}$$

61. $P(\text{strike or one}) = P(\text{strike}) + P(\text{one})$
$$= \frac{n(\text{strike}) + n(\text{one})}{n(S)}$$
$$= \frac{3+1}{8} = \frac{4}{8} = \frac{1}{2}$$

62. $P(100 \text{ or } 30) = P(100) + P(30)$
$$= \frac{n(100) + n(30)}{n(S)}$$
$$= \frac{1+1}{20} = \frac{2}{20} = \frac{1}{10}$$

63. There are 34 households out of 100 with an income of \$75,000 or more.
$$P(E) = \frac{n(E)}{n(S)} = \frac{n(75{,}000 \text{ or more})}{n(\text{total households})} = \frac{34}{100} = \frac{17}{50}$$

64. There are 42 households out of 100 with an income between \$25,000 and \$74,999.
$$P(E) = \frac{n(E)}{n(S)} = \frac{n(25000 \text{ to } 74999)}{n(\text{total households})} = \frac{42}{100} = \frac{21}{50}$$

65. There are 48 households out of 100 with an income of less than \$50,000.
$$P(E) = \frac{n(E)}{n(S)} = \frac{n(\text{less than } 50{,}000)}{n(\text{total households})} = \frac{48}{100} = \frac{12}{25}$$

66. There are 52 households out of 100 with an income of \$50,000 or more.
$$P(E) = \frac{n(E)}{n(S)} = \frac{n(\$50{,}000 \text{ or more})}{n(\text{total households})} = \frac{52}{100} = \frac{13}{25}$$

67. **a.** $P(1 \text{ or } 2) = P(1) + P(2) = 0.34 + 0.37 = 0.71$

b. $P(1 \text{ or more}) = 1 - P(\text{none}) = 1 - 0.09 = 0.91$

c. $P(3 \text{ or fewer}) = 1 - P(4 \text{ or more})$
$= 1 - 0.06 = 0.94$

d. $P(3 \text{ or more}) = P(3) + P(4 \text{ or more})$
$= 0.14 + 0.06 = 0.20$

e. $P(\text{fewer than } 2) = P(0) + P(1)$
$= 0.09 + 0.34 = 0.43$

f. $P(\text{fewer than } 1) = P(0) = 0.09$

g. $P(1, 2, \text{ or } 3) = P(1) + P(2) + P(3)$
$= 0.34 + 0.37 + 0.14 = 0.85$

h. $P(2 \text{ or more}) = P(2) + P(3) + P(4 \text{ or more})$
$= 0.37 + 0.14 + 0.06 = 0.57$

68. **a.** $P(\text{at most } 2) = P(0) + P(1) + P(2)$
$= 0.10 + 0.15 + 0.20 = 0.45$

b. $P(\text{at least } 2) = P(2) + P(3) + P(4 \text{ or more})$
$= 0.20 + 0.24 + 0.31 = 0.75$

c. $P(\text{at least } 1) = 1 - P(0) = 1 - 0.10 = 0.90$

69. **a.** $P(\text{freshman or female})$
$= P(\text{freshman}) + P(\text{female}) - P(\text{freshman and female})$
$$= \frac{n(\text{freshman}) + n(\text{female}) - n(\text{freshman and female})}{n(S)}$$
$$= \frac{18+15-8}{33} = \frac{25}{33}$$

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b. $P(\text{sophomore or male})$

$= P(\text{sophomore}) + P(\text{male}) - P(\text{sophomore and male})$

$= \dfrac{n(\text{sophomore}) + n(\text{male}) - n(\text{sophomore and male})}{n(S)}$

$= \dfrac{15+18-8}{33} = \dfrac{25}{33}$

70. a. $P(\text{female or under 40})$

$= P(\text{female}) + P(\text{under 40}) - P(\text{female and under 40})$

$= \dfrac{n(\text{female}) + n(\text{under 40}) - n(\text{female and under 40})}{n(S)}$

$= \dfrac{4+5-2}{13} = \dfrac{7}{13}$

b. $P(\text{male or over 40})$

$= P(\text{male}) + P(\text{over 40}) - P(\text{male and over 40})$

$= \dfrac{n(\text{male}) + n(\text{over 40}) - n(\text{male and over 40})}{n(S)}$

$= \dfrac{9+8-6}{13} = \dfrac{11}{13}$

71. $P(\text{at least 2 with same birthday})$

$= 1 - P(\text{none with same birthday})$

$= 1 - \dfrac{n(\text{different birthdays})}{n(S)}$

$= 1 - \dfrac{365 \cdot 364 \cdot 363 \cdot 362 \cdot 361 \cdot 360 \cdots 354}{365^{12}}$

$\approx 1 - 0.833$

$= 0.167$

72. $P(\text{at least 2 with same birthday})$

$= 1 - P(\text{none with same birthday})$

$= 1 - \dfrac{n(\text{different birthdays})}{n(S)}$

$= 1 - \dfrac{365 \cdot 364 \cdot 363 \cdot 362 \cdot 361 \cdot 360 \cdots 331}{365^{35}}$

$\approx 1 - 0.186$

$= 0.814$

73. The number of different selections of 6 numbers is the number of ways we can choose 5 white balls and 1 red ball, where the order of the white balls is not important. This requires the use of the Multiplication Principle and the combination formula. Thus, the total number of distinct ways to pick the 6 numbers is given by

$n(\text{white balls}) \cdot n(\text{red ball})$

$= C(59,5) \cdot C(35,1)$

$= \dfrac{59!}{5!(59-5)!} \cdot \dfrac{35!}{1!(35-1)!}$

$= \dfrac{59!}{5!\,54!} \cdot \dfrac{35!}{1!\,34!}$

$= \left(\dfrac{59 \cdot 58 \cdot 57 \cdot 56 \cdot 55}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}\right)\left(\dfrac{35 \cdot 34!}{34!}\right)$

$= \dfrac{59 \cdot 58 \cdot 57 \cdot 56 \cdot 55 \cdot 35}{5 \cdot 4 \cdot 3 \cdot 2}$

$= 175,223,510$

Since each possible combination is equally likely, the probability of winning on a \$2 play is

$P(\text{win on \$2 play}) = \dfrac{1}{175,223,510}$

≈ 0.00000000571

74. 2; left; 3; down

75. $x = 6\cos\left(\dfrac{2\pi}{3}\right) = -3$

$y = 6\sin\left(\dfrac{2\pi}{3}\right) = 3\sqrt{3}$

76. $\log_5(x+3) = 2$

$5^2 = x+3$

$25 = x+3$

$x = 22$

The solution set is $\{22\}$.

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77. $\begin{cases} 3x + y + 2z = 1 \\ 2x - 2y + 5z = 5 \\ x + 3y + 2z = -9 \end{cases}$

Write the augmented matrix:

$$\left[\begin{array}{ccc|c} 3 & 1 & 2 & 1 \\ 2 & -2 & 5 & 5 \\ 1 & 3 & 2 & -9 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 2 & -9 \\ 2 & -2 & 5 & 5 \\ 3 & 1 & 2 & 1 \end{array}\right] \quad (R_1 \leftrightarrow R_3)$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 2 & -9 \\ 0 & 1 & -\frac{1}{8} & -\frac{23}{8} \\ 0 & -8 & -4 & 28 \end{array}\right] \quad \left(R_2 = -\frac{1}{8} r_1\right)$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & \frac{19}{8} & -\frac{3}{8} \\ 0 & 1 & -\frac{1}{8} & -\frac{23}{8} \\ 0 & 0 & -5 & 5 \end{array}\right] \quad \begin{pmatrix} R_3 = 8r_2 + r_3 \\ R_1 = -3r_2 + r_1 \end{pmatrix}$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & \frac{19}{8} & -\frac{3}{8} \\ 0 & 1 & -\frac{1}{8} & -\frac{23}{8} \\ 0 & 0 & 1 & -1 \end{array}\right] \quad \left(R_3 = -\frac{1}{5} r_3\right)$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -1 \end{array}\right] \quad \begin{pmatrix} R_2 = \frac{1}{8} r_3 + r_2 \\ R_1 = -\frac{19}{8} r_3 + r_2 \end{pmatrix}$$

The solution is $x = 2$, $y = -3$, $z = -1$ or $(2, -3, -1)$.

Chapter 13 Review Exercises

1. $\varnothing, \{\text{Dave}\}, \{\text{Joanne}\}, \{\text{Erica}\}, \{\text{Dave, Joanne}\}, \{\text{Dave, Erica}\}, \{\text{Joanne, Erica}\}, \{\text{Dave, Joanne, Erica}\}$

2. $n(A) = 8,\ n(B) = 12,\ n(A \cap B) = 3$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 8 + 12 - 3 = 17$$

3. $n(A) = 12,\ n(A \cup B) = 30,\ n(A \cap B) = 6$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$30 = 12 + n(B) - 6$$
$$n(B) = 30 - 12 + 6 = 24$$

4. From the figure: $n(A) = 20 + 2 + 6 + 1 = 29$

5. From the figure:
$n(A \text{ or } B) = 20 + 2 + 6 + 1 + 5 + 0 = 34$

6. From the figure:
$n(A \text{ and } C) = n(A \cap C) = 1 + 6 = 7$

7. From the figure:
$n(\text{not in } B) = 20 + 1 + 4 + 20 = 45$

8. From the figure:
$n(\text{neither in } A \text{ nor in } C) = n(\overline{A \cup C}) = 20 + 5 = 25$

9. From the figure: $n(\text{in } B \text{ but not in } C) = 2 + 5 = 7$

10. $P(8,3) = \dfrac{8!}{(8-3)!} = \dfrac{8!}{5!} = \dfrac{8 \cdot 7 \cdot 6 \cdot 5!}{5!} = 336$

11. $C(8,3) = \dfrac{8!}{(8-3)!\,3!} = \dfrac{8!}{5!\,3!} = \dfrac{8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 3 \cdot 2 \cdot 1} = 56$

12. There are 2 choices of material, 3 choices of color, and 10 choices of size. The complete assortment would have: $2 \cdot 3 \cdot 10 = 60$ suits.

13. There are two possible outcomes for each game or $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^7 = 128$ outcomes for 7 games.

14. Since order is significant, this is a permutation.

$$P(9,4) = \frac{9!}{(9-4)!} = \frac{9!}{5!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5!} = 3024$$

ways to seat 4 people in 9 seats.

15. Choose 4 runners –order is significant:

$$P(8,4) = \frac{8!}{(8-4)!} = \frac{8!}{4!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4!} = 1680$$

ways a squad can be chosen.

16. Choose 2 teams from 14–order is not significant:

$$C(14,2) = \frac{14!}{(14-2)!\,2!} = \frac{14!}{12!\,2!} = \frac{14 \cdot 13 \cdot 12!}{12! \cdot 2 \cdot 1} = 91$$

ways to choose 2 teams.

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17. There are $8 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 2 = 1,600,000$ possible phone numbers.

18. There are $24 \cdot 9 \cdot 10 \cdot 10 \cdot 10 = 216,000$ possible license plates.

19. There are two choices for each digit, so there are $2^8 = 256$ different numbers. (Note this allows numbers with initial zeros, such as 011.)

20. Since there are repeated colors:
$$\frac{10!}{4! \cdot 3! \cdot 2! \cdot 1!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1 \cdot 1} = 12,600$$
different vertical arrangements.

21. **a.** $C(9,4) \cdot C(9,3) \cdot C(9,2) = 126 \cdot 84 \cdot 36 = 381,024$
committees can be formed.

b. $C(9,4) \cdot C(5,3) \cdot C(2,2) = 126 \cdot 10 \cdot 1 = 1260$
committees can be formed.

22. **a.** $365 \cdot 364 \cdot 363 \cdots 348 = 8.634628387 \times 10^{45}$

b. $P(\text{no one has same birthday}) = \dfrac{365 \cdot 364 \cdot 363 \cdots 348}{365^{18}} \approx 0.6531$

c. $P(\text{at least 2 have same birthday}) = 1 - P(\text{no one has same birthday})$
$= 1 - 0.6531 = 0.3469$

23. **a.** $P(\text{unemployed}) = 0.062$

b. $P(\text{not unemployed}) = 1 - P(\text{unemployed}) = 1 - 0.062 = 0.938$

24. $P(\$1 \text{ bill}) = \dfrac{n(\$1 \text{ bill})}{n(S)} = \dfrac{4}{9}$

25. Let S be all possible selections, so $n(S) = 100$. Let D be a card that is divisible by 5, so $n(D) = 20$. Let PN be a card that is 1 or a prime number, so $n(PN) = 26$.

$$P(D) = \frac{n(D)}{n(S)} = \frac{20}{100} = \frac{1}{5} = 0.2$$

$$P(PN) = \frac{n(PN)}{n(S)} = \frac{26}{100} = \frac{13}{50} = 0.26$$

26. Let S be all possible selections, let T be a car that needs a tune-up, and let B be a car that needs a brake job.

a. $P(\text{Tune-up or Brake job}) = P(T \cup B) = P(T) + P(B) - P(T \cap B) = 0.6 + 0.1 - 0.02 = 0.68$

b. $P(\text{Tune-up but not Brake job}) = P(\text{Tune-up}) - P(\text{Tune-up and Brake job}) = P(T) - P(T \cap B) = 0.6 - 0.02 = 0.58$

c. $P(\text{Neither Tune-up nor Brake job}) = 1 - P(\text{Tune-up or Brake job}) = 1 - (P(T) + P(B) - P(T \cap B)) = 1 - (0.6 + 0.1 - 0.02) = 0.32$

Chapter 13 Test

1. From the figure:
$n(\text{physics}) = 4 + 2 + 7 + 9 = 22$

2. From the figure:
$n(\text{biology } or \text{ chemistry } or \text{ physics}) = 22 + 8 + 2 + 4 + 9 + 7 + 15 = 67$
Therefore,
$n(\text{none of the three}) = 70 - 67 = 3$

3. From the figure:
$n(\text{only biology } and \text{ chemistry}) = n(\text{biol. } and \text{ chem.}) - n(\text{biol. } and \text{ chem. } and \text{ phys.}) = (8 + 2) - 2 = 8$

4. From the figure:
$n(\text{physics } or \text{ chemistry}) = 4 + 2 + 7 + 9 + 15 + 8 = 45$

5. $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$

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6. $P(10,6) = \dfrac{10!}{(10-6)!} = \dfrac{10!}{4!}$

$= \dfrac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4!}{4!}$

$= 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5$

$= 151{,}200$

7. $C(11,5) = \dfrac{11!}{5!(11-5)!} = \dfrac{11!}{5!6!}$

$= \dfrac{11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6!}{5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 6!}$

$= \dfrac{11\cdot 10\cdot 9\cdot 8\cdot 7}{5\cdot 4\cdot 3\cdot 2\cdot 1}$

$= 462$

8. Since the order in which the colors are selected doesn't matter, this is a combination problem. We have $n = 21$ colors and we wish to select $r = 6$ of them.

$C(21,6) = \dfrac{21!}{6!(21-6)!} = \dfrac{21!}{6!15!}$

$= \dfrac{21\cdot 20\cdot 19\cdot 18\cdot 17\cdot 16\cdot 15!}{6!15!}$

$= \dfrac{21\cdot 20\cdot 19\cdot 18\cdot 17\cdot 16}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}$

$= 54{,}264$

There are 54,264 ways to choose 6 colors from the 21 available colors.

9. Because the letters are not distinct and order matters, we use the permutation formula for non-distinct objects. We have four different letters, two of which are repeated (E four times and D two times).

$\dfrac{n!}{n_1!n_2!n_3!n_4!} = \dfrac{8!}{4!2!1!1!}$

$= \dfrac{8\cdot 7\cdot 6\cdot 5\cdot 4!}{4!\cdot 2\cdot 1}$

$= \dfrac{8\cdot 7\cdot 6\cdot 5}{2}$

$= 4\cdot 7\cdot 6\cdot 5$

$= 840$

There are 840 distinct arrangements of the letters in the word REDEEMED.

10. Since the order of the horses matters and all the horses are distinct, we use the permutation formula for distinct objects.

$P(8,2) = \dfrac{8!}{(8-2)!} = \dfrac{8!}{6!} = \dfrac{8\cdot 7\cdot 6!}{6!} = 8\cdot 7 = 56$

There are 56 different exacta bets for an 8-horse race.

11. We are choosing 3 letters from 26 distinct letters and 4 digits from 10 distinct digits. The letters and numbers are placed in order following the format LLL DDDD with repetitions being allowed. Using the Multiplication Principle, we get $26\cdot 26\cdot 23\cdot 10\cdot 10\cdot 10\cdot 10 = 155{,}480{,}000$

Note that there are only 23 possibilities for the third letter. There are 155,480,000 possible license plates using the new format.

12. Let A = Kiersten accepted at USC, and B = Kiersten accepted at FSU. Then, we get $P(A) = 0.60$, $P(B) = 0.70$, and $P(A \cap B) = 0.35$.

a. Here we need to use the Addition Rule.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.60 + 0.70 - 0.35$

$= 0.95$

Kiersten has a 95% chance of being admitted to at least one of the universities.

b. Here we need the Complement of an event.

$P(\overline{B}) = 1 - P(B) = 1 - 0.70 = 0.30$

Kiersten has a 30% chance of not being admitted to FSU.

13. a. Since the bottle is chosen at random, all bottles are equally likely to be selected. Thus,

$P(\text{Coke}) = \dfrac{5}{8+5+4+3} = \dfrac{5}{20} = \dfrac{1}{4} = 0.25$

There is a 25% chance that the selected bottle contains Coke.

b. $P(\text{Pepsi} \cup \text{IBC}) = \dfrac{8+3}{8+5+4+3} = \dfrac{11}{20} = 0.55$

There is a 55% chance that the selected bottle contains either Pepsi or IBC.

14. Since the ages cover all possibilities and the age groups are mutually exclusive, the sum of all the probabilities must equal 1.

$0.03 + 0.23 + 0.29 + 0.25 + 0.01 = 0.81$

$1 - 0.81 = 0.19$

The given probabilities sum to 0.81. This means the missing probability (for 18-20) must be 0.19.

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15. The sample space for picking 5 out of 10 numbers in a particular order contains

$$P(10,5)=\frac{10!}{(10-5)!}=\frac{10!}{5!}=30,240 \text{ possible}$$

outcomes. One of these is the desired outcome. Thus, the probability of winning is:

$$P(E)=\frac{n(E)}{n(S)}=\frac{n(\text{winning})}{n(\text{total possible outcomes})}$$

$$=\frac{1}{30,240}\approx 0.000033069$$

16. The number of elements in the sample space can be obtained by using the Multiplication Principle:

$6\cdot 6\cdot 6\cdot 6\cdot 6=7,776$

Consider the rolls as a sequence of 5 slots. The number of ways to position 2 fours in 5 slots is $C(5,2)$. The remaining three slots can be filled with any of the five remaining numbers from the die. Repetitions are allowed so this can be done in $5\cdot 5\cdot 5=125$ different ways.

Therefore, the total number of ways to get exactly 2 fours is

$$C(5,2)\cdot 125=\frac{5!}{2!\,3!}\cdot 125=\frac{5\cdot 4\cdot 125}{2}=1250$$

The probability of getting exactly 2 fours on 5 rolls of a die is given by

$$P(\text{exactly 2 fours})=\frac{1250}{7776}=\frac{625}{3888}\approx 0.1608.$$

Chapter 13 Cumulative Review

1. $3x^2-2x=-1$

$3x^2-2x+1=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-2)\pm\sqrt{(-2)^2-4(3)(1)}}{2(3)}$$

$$=\frac{2\pm\sqrt{4-12}}{6}=\frac{2\pm\sqrt{-8}}{6}$$

$$=\frac{2\pm 2\sqrt{2}i}{6}=\frac{1\pm\sqrt{2}i}{3}$$

The solution set is $\left\{\frac{1-\sqrt{2}i}{3},\frac{1+\sqrt{2}i}{3}\right\}$.

2. $f(x)=x^2+4x-5$

$a=1, b=4, c=-5$. Since $a=1>0$, the graph opens up. The x-coordinate of the vertex is

$$x=-\frac{b}{2a}=-\frac{4}{2(1)}=-2.$$

The y-coordinate of the vertex is

$$f\left(-\frac{b}{2a}\right)=f(-2)=(-2)^2+4(-2)-5.$$

$$=4-8-5=-9$$

Thus, the vertex is $(-2,-9)$. The axis of symmetry is the line $x=-2$ .The discriminant is:

$b^2-4ac=(4)^2-4(1)(-5)=16+20=36>0$.

So the graph has two x-intercepts.

The x-intercepts are found by solving:

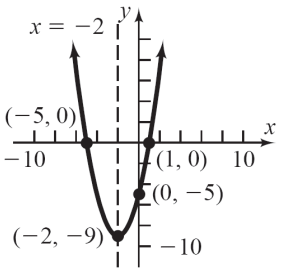
$x^2+4x-5=0$

$(x+5)(x-1)=0$

$x=-5$ or $x=1$

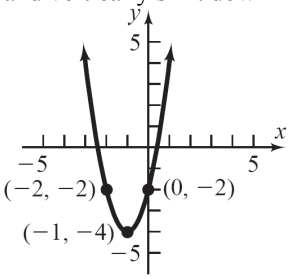
The x-intercepts are -5 and 1.

The y-intercept is $f(0)=(0)^2+4\cdot(0)-5=-5$.



3. $y=2(x+1)^2-4$

Using the graph of $y=x^2$, horizontally shift to the left 1 unit, vertically stretch by a factor of 2, and vertically shift down 4 units.



4. $|x-4|\le 0.01$

$-0.01\le x-4\le 0.01$

$-0.01+4\le x\le 0.01+4$

$3.99\le x\le 4.01$

The solution set is

$\{x\,|\,3.99\le x\le 4.01\}$ or $[3.99,4.01]$

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5. $f(x) = 5x^4 - 9x^3 - 7x^2 - 31x - 6$

Step 1: $f(x)$ has at most 4 real zeros.

Step 2: Possible rational zeros:

$p = \pm 1, \pm 2, \pm 3, \pm 6; \quad q = \pm 1, \pm 5;$

$$\frac{p}{q} = \pm 1, \pm\frac{1}{5}, \pm 2, \pm\frac{2}{5}, \pm 3, \pm\frac{3}{5}, \pm 6, \pm\frac{6}{5}$$

Step 3: Using the Bounds on Zeros Theorem:

$f(x) = 5\left(x^4 - 1.8x^3 - 1.4x^2 - 6.2x - 1.2\right)$

$a_3 = -1.8,\ a_2 = -1.4,\ a_1 = -6.2,\ a_0 = -1.2$

Max $\{1, |-1.2| + |-6.2| + |-1.4| + |-1.8|\}$

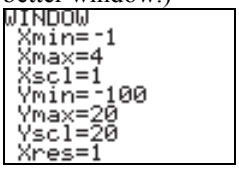
$= \text{Max}\ \{1, 10.6\} = 10.6$

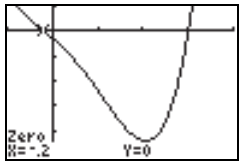
$1 + \text{Max}\ \{|-1.2|, |-6.2|, |-1.4|, |-1.8|\}$

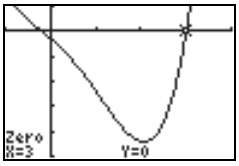
$= 1 + 6.2 = 7.2$

The smaller of the two numbers is 7.2. Thus, every zero of f lies between –7.2 and 7.2.

Graphing using the bounds: (Second graph has a better window.)







Step 4: From the graph we see that there are x-intercepts at -0.2 and 3. Using synthetic division with 3:

3) 5	−9	−7	−31	−6
	15	18	33	6
5	6	11	2	0

Since the remainder is 0, $x - 3$ is a factor. The other factor is the quotient: $5x^3 + 6x^2 + 11x + 2$.

Using synthetic division with 2 on the quotient:

−0.2) 5	6	11	2
	−1	−1	−2
5	5	10	0

Since the remainder is 0, $x - (-0.2) = x + 0.2$ is a factor. The other factor is the quotient: $5x^2 + 5x + 10 = 5\left(x^2 + x + 2\right)$.

Factoring, $f(x) = 5(x^2 + x + 2)(x - 3)(x + 0.2)$

The real zeros are 3 and -0.2.

The complex zeros come from solving $x^2 + x + 2 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1-8}}{2} = \frac{-1 \pm \sqrt{-7}}{2}$$

$$= \frac{-1 \pm \sqrt{7}i}{2}$$

Therefore, over the set of complex numbers, $f(x) = 5x^4 - 9x^3 - 7x^2 - 31x - 6$ has zeros $\left\{-\frac{1}{2} + \frac{\sqrt{7}}{2}i, -\frac{1}{2} - \frac{\sqrt{7}}{2}i, -\frac{1}{5}, 3\right\}$.

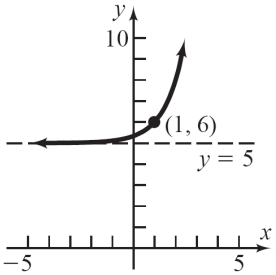
6. $g(x) = 3^{x-1} + 5$

Using the graph of $y = 3^x$, shift the graph horizontally 1 unit to the right, then shift the graph vertically 5 units upward.

Domain: All real numbers or $(-\infty, \infty)$

Range: $\{y \mid y > 5\}$ or $(5, \infty)$

Horizontal Asymptote: $y = 5$



7. $\log_3(9) = \log_3(3^2) = 2$

8. $\log_2(3x - 2) + \log_2 x = 4$

$\log_2\left(x(3x - 2)\right) = 4$

$x(3x - 2) = 2^4$

$3x^2 - 2x = 16$

$3x^2 - 2x - 16 = 0$

$(3x - 8)(x + 2) = 0$

$x = \frac{8}{3}$ or $x = -2$

Since $x = -2$ makes the original logarithms undefined, the solution set is $\left\{\frac{8}{3}\right\}$.

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9. Multiply each side of the first equation by –3 and add to the second equation to eliminate x; multiply each side of the first equation by 2 and add to the third equation to eliminate x:

$$\begin{cases} x - 2y + z = 15 \\ 3x + y - 3z = -8 \\ -2x + 4y - z = -27 \end{cases}$$

$$\begin{array}{r} -3x + 6y - 3z = -45 \\ 3x + y - 3z = -8 \\ \hline 7y - 6z = -53 \end{array}$$

$$\begin{array}{rcl} x - 2y + z = 15 & \xrightarrow{2} & 2x - 4y + 2z = 30 \\ -2x + 4y - z = -27 & \longrightarrow & \underline{-2x + 4y - z = -27} \\ & & z = 3 \end{array}$$

Substituting and solving for the other variables:

$$z = 3 \Rightarrow 7y - 6(3) = -53$$
$$7y = -35$$
$$y = -5$$
$$z = 3, y = -5 \Rightarrow x - 2(-5) + 3 = 15$$
$$x + 10 + 3 = 15 \Rightarrow x = 2$$

The solution is $x = 2,\ y = -5,\ z = 3$ or $(2, -5, 3)$.

10. –3, 1, 5, 9, ... is an arithmetic sequence with $a = -3,\ d = 4$.

Using $a_n = a + (n-1)d$,

$$a_{33} = -3 + (33-1)\cdot 4$$
$$= -3 + 32 \cdot 4$$
$$= -3 + 128$$
$$= 125$$

To compute the sum of the first 20 terms, we use $S_{20} = \frac{20}{2}(a + a_{20})$.

$$a_{20} = -3 + (20-1)\cdot 4$$
$$= -3 + 19 \cdot 4$$
$$= -3 + 76$$
$$= 73$$

Therefore,

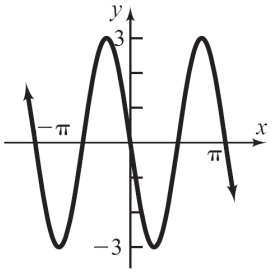
$$S_{20} = \frac{20}{2}(a + a_{20})$$
$$= \frac{20}{2}(-3 + 73)$$
$$= 10 \cdot 70$$
$$= 700.$$

11. $y = 3\sin(2x + \pi) = 3\sin\left(2\left(x + \frac{\pi}{2}\right)\right)$

Amplitude: $|A| = |3| = 3$

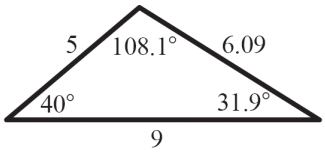
Period: $T = \frac{2\pi}{2} = \pi$

Phase Shift: $\frac{\phi}{\omega} = \frac{-\pi}{2} = -\frac{\pi}{2}$



12. Use the Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = 5^2 + 9^2 - 2\cdot 5 \cdot 9\cos 40^\circ$$
$$= 106 - 90\cos 40^\circ$$
$$a \approx 6.09$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \approx \frac{6.09^2 + 9^2 - 5^2}{2(6.09)(9)} = \frac{93.0881}{109.62}$$
$$B \approx \cos^{-1}\left(\frac{93.0881}{109.62}\right) \approx 31.9^\circ$$
$$C = 180^\circ - A - B \approx 180^\circ - 40^\circ - 31.9^\circ = 108.1^\circ$$



Area of the triangle $= \frac{1}{2}\cdot 5 \cdot 9 \cdot \sin(40^\circ) \approx 14.46$ square units.

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Chapter 13 Projects

Project I

1. Table 3 is a probability model since the total of the probabilities is 1.

Cash Prize	Probability
Jackpot	0.00000000386
$1,000,000	0.00000005408
$5000	0.00000135192
$500	0.00001892689
$50	0.00009328256
$5	0.00342036036
$2	0.01770813839
$1	0.04674948535
$0	0.93200839659
Total	1.00000000000

2. There are 75 numbers to choose for the first 5 'white' numbers and 15 numbers to choose for the last 'gold' number. Thus there are ${}_{75}C_5$ possibilities for the 'white' numbers and ${}_{15}C_1$ possibilities for the 'gold' number. Multiply these together to find the total possibilities.

$$\begin{aligned} {}_{75}C_5 \cdot {}_{15}C_1 &= (17259390)(15) \\ &= 258,890,850 \end{aligned}$$

So the probability of winning is:

$$\begin{aligned} P(\text{win}) &= \frac{1}{258,890,850} \\ &\approx 0.00000000386 \end{aligned}$$

3. To calculate the expected value, multiply each numeric outcome by its corresponding probability and then add these products.

Cash Prize	Probability	prize · prob
$20,000,000	0.00000000386	0.07720000000
$1,000,000	0.00000005408	0.05408000000
$5000	0.00000135192	0.00675960000
$500	0.00001892689	0.00946344500
$50	0.00009328256	0.00466412800
$5	0.00342036036	0.01710180180
$2	0.01770813839	0.03541627678
$1	0.04674948535	0.04674948535
$0	0.93200839659	0.00000000000
expect value		0.25143473693

So the expected cash prize is 0.25.

4. The expected financial result from purchasing one ticket is $\$0.25 - \$1.00 = -\$0.75$. Therefore, your expected profit from one ticket is $-\$0.75$.

5. Use the same procedure replacing the Jackpot with $100,000,000.

Cash Prize	Probability	prize · prob
$100,000,000	0.00000000386	0.38600000000
$1,000,000	0.00000005408	0.05408000000
$5000	0.00000135192	0.00675960000
$500	0.00001892689	0.00946344500
$50	0.00009328256	0.00466412800
$5	0.00342036036	0.01710180180
$2	0.01770813839	0.03541627678
$1	0.04674948535	0.04674948535
$0	0.93200839659	0.00000000000
expect value		0.56023473693

The expected financial result from purchasing one ticket is $\$0.56 - \$1.00 = -\$0.44$. Therefore, your expected profit from one ticket is $-\$0.44$.

6. We need to solve the expected value equation for the Jackpot amount that would make the expected value equal to $1.00. Thus:

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Cash Prize	Probability	prize · prob
JP	0.00000000386	$J(0.00000000386)$
$1,000,000	0.00000005408	0.05408000000
$5000	0.00000135192	0.00675960000
$500	0.00001892689	0.00946344500
$50	0.00009328256	0.00466412800
$5	0.00342036036	0.01710180180
$2	0.01770813839	0.03541627678
$1	0.04674948535	0.04674948535
$0	0.93200839659	0.00000000000
expect value		1.00000000000

$$J(0.00000000386) + 0.05408 + 0.0067596 + 0.009463445 + 0.004664128 + 0.0171018018 + 0.03541627678 + 0.04674948535 = 1$$

$$J(0.00000000386) + 0.17423473693 = 1$$

$$J(0.00000000386) = 0.82576526307$$

$$J \approx \$213,928,825$$

7. Answers will vary.

Project II

1. 0 bit errors: 1011

1 bit errors: 0011, 1111, 1001, 1010

2 bit errors: 0111, 0001, 0010, 1101, 1110, 1000

3 bit errors: 0110, 0101, 0000, 1100

4 bit errors: 0100

2. $\mathrm{P}(\text{symbol received correctly}) = \left(\frac{2}{3}\right)^4 = \frac{16}{81}$

3. # of received symbols with 2 bit errors:
$C(8,2) = 28$

$$\mathrm{P}(\text{received correctly}) = \left(\frac{2}{3}\right)^8 = \frac{256}{6561}$$

$$\mathrm{P}(\text{received incorrectly}) = 1 - \mathrm{P}(\text{received correctly}) = \frac{6305}{6561}$$

4. Let k = # of errors, n = 8 = length of symbol.
Probability of k errors :

$$P(n,k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

$$P(8,k) = \binom{8}{k}\left(\frac{1}{3}\right)^k\left(\frac{2}{3}\right)^{8-k}$$

Since this parity code only detects odd numbers of errors,

$$P(\text{error detected}) = P(8,1) + P(8,3) + P(8,5) + P(8,7)$$

$$= \binom{8}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^7 + \binom{8}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^5 + \binom{8}{5}\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^3 + \binom{8}{7}\left(\frac{1}{3}\right)^7\left(\frac{2}{3}\right)^1$$

$$= 0.156464 + 0.272992 + 0.068044 + 0.002423$$

$$= 0.499923$$

To find the probability that an error occurred but is not detected, we need to assume that an even number of errors occurred:

$$P(\text{error occured, but not detected}) = P(8,2) + P(8,4) + P(8,6) + P(8,8)$$

$$= \binom{8}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^6 + \binom{8}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^4 + \binom{8}{6}\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)^2 + \binom{8}{8}\left(\frac{1}{3}\right)^8\left(\frac{2}{3}\right)^0$$

$$= 0.273402 + 0.170364 + 0.016985 + 0.000151$$

$$= 0.460951$$

Project III

Answers will vary.

Project IV

e. Answers will vary, depending on the L_2 generated by the calculator.

f. The data accumulates around $y = 0.5$.

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Project V

One simulation might be:

Woman has	Woman told you about	Probability
Boy-Boy	Older boy	$\frac{1}{4}$
Boy-Boy	Younger boy	$\frac{1}{4}$
Boy-Girl	Younger boy	$\frac{1}{4}$
Girl-Boy	Older boy	$\frac{1}{4}$

We leave out the combinations where she would have to tell you about a girl. Thus, the probability that she has 2 boys is $\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$.

Man has	Man told you about	Probability
Boy-Boy	Older boy	$\frac{1}{2}$
Girl-Boy	Older boy	$\frac{1}{2}$

Thus the probability he has two boys is $\frac{1}{2}$. The probabilities are the same.

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