

Operations with Functions

Given $f(x)$ and $g(x) \dots$

Operation

$$(f + g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

$$(f \cdot g)(x) = f(x)g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Note: $g(x) \neq 0$

Example:

$$f(x) = \sqrt{3x} \quad g(x) = \sqrt{4-x^2}$$

Find $(f \cdot g)(x)$

$$\begin{aligned} &= f(x) \cdot g(x) \\ &= \sqrt{3x} (\sqrt{4-x^2}) \\ &= \sqrt{12x-3x^2} \end{aligned}$$

Find $(\frac{f}{g})(x)$.

$$\begin{aligned} &= \frac{\sqrt{3x}}{\sqrt{4-x^2}} \\ &= \frac{\sqrt{3x}}{\sqrt{4-x^2}} \cdot \frac{\sqrt{4-x^2}}{\sqrt{4-x^2}} \\ &= \frac{\sqrt{12x-3x^2}}{4-x^2} \end{aligned}$$

Composition of Functions

$$(f \circ g)(x) = f(g(x)) \quad \text{Read "f of g of x"}$$

Example:

$$f(x) = 2x^2 + x \quad g(x) = 3x - 5$$

1) $(f \circ g)(2)$ (or $f(g(2))$)
first find $g(2)$. *Do the inside 1st
 $g(2) = 3(2) - 5 = \boxed{1} \rightarrow$ Now find $f(1)$.
 $f(1) = 2(1)^2 + 1$
 $= 2 + 1$
 $= \boxed{3}$

2) $(f \circ g)(x)$ (or $f(g(x))$)
Substitute " $g(x)$ " into every " x " in $f(x)$ and simplify
 $(f \circ g)(x) = 2(3x-5)^2 + 3x - 5$
 $= 2(9x^2 - 30x + 25) + 3x - 5$
 $= 18x^2 - 60x + 50 + 3x - 5$
 $= \boxed{18x^2 - 57x + 45}$

3) $(g \circ f)(x)$ (or $g(f(x))$)
Substitute " $f(x)$ " into every " x " in $g(x)$ and simplify
 $(g \circ f)(x) = 3(2x^2 + x) - 5$
 $= \boxed{6x^2 + 3x - 5}$