

## Probability WS 1- COUNTING

**Determine the correct number of possibilities. Show all your work.**

1. A student must write a report on any two books out of a list of 8 books for an English class. How many combinations of reports are possible?
2. Three class officers, president, secretary and treasurer are to be selected by the student council from 25 student representatives. How many different slates of candidates are possible?
3. How many integers greater than or equal to 1000 but less than 10,000 contain no 5's?
4. In how many ways can 4 people be seated in 6 chairs?
5. The coach has 12 players on the basketball team, but he can only take 10 to IASAS. How many different ways can the team be selected (without regard to position)?
6. IASAS rules state that in any IASAS tournament, each school must play every other school. How many games does that necessitate?
7. In the S-league there are 9 teams and each team has to play every other team twice. How many games will have to be played?
8. Eight people apply for three job openings. In how many ways can the 3 openings be filled if the jobs are:
  - a) All different?
  - b) All the same kind?
9. Mr. Baker has a collection of 20 essay questions on the Civil War. If he makes up a test with 5 questions, how many different tests are possible?
10. On the student council there are 5 representatives from each class. A committee of 4 is needed for next year's Christmas Ball. How many different committees are possible if:
  - a) There is a representative from each class?
  - b) There are 2 seniors and 2 underclassmen (9, 10 or 11)?
  - c) There is one senior and the remaining committee members are underclassmen?
11. Consider a standard deck of 52 cards. Determine the number of distinct six card hands that are possible which include:

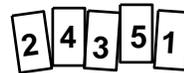
a) no restrictions	b) only clubs	c) 2 clubs and 4 diamonds
d) no sevens	e) 4 kings	f) only 1 jack and 4 queens

12. Using only Aces, Kings, Jacks and Queens from a normal deck of 52 cards, how many different 5 card hands are possible?
13. License plates in Singapore contain 3 letters followed by 4 numbers. An eye witness to a hit and run accident can remember only that the plate started with the letters SB and the last number was a 4. How many plates with this arrangement are possible?
14. Three identical door prizes are to be given to three lucky people in a crowd of 100. In how many ways can this be done?
15. Baskin Robbins has 20 flavors of ice cream and 11 flavors of sherbet. In how many ways could you select:
  - a) A scoop of ice cream and then a scoop of sherbet?
  - b) A scoop of ice cream or a scoop of sherbet?
16. There are 3 roads from town A to town B, 5 roads from town B to town C and 4 roads from town C to town D. How many different ways are there to go from A to D via B and C? How many different round trips are possible?
17. A teacher must pick 3 high school students from a class of 30 to prepare and serve food at the junior high school picnic. How many choices are possible?
18. How many ways can 8 jackets of different styles be hung, on a straight bar? On a circular track?
19. A basketball team has 4 guards, 5 forwards and 3 centers on its roster. In how many ways can a team be created with 1 right guard, 1 left guard, 1 right forward, 1 left forward and 1 center?
20. A bookshelf has space for 5 books. If 7 books are available, how many different arrangements can be made on the bookshelf?
21. A hockey team consists of 10 Canadians, 2 Americans and 6 Europeans they can each play all positions, the coach selects 6 players for the starting lineup. How many ways can the coach select 4 Canadians, 1 American and 1 European?
22. A town council consists of 8 members including the mayor
  - a) How many different committees of 4 can be chosen from this council?
  - b) How many of these committees include the mayor?
  - c) How many of these committees do not include the mayor?

## Probability WS 2

Show all your work.

- In a drawer are 6 white socks and 2 black socks. In the dark you reach in and pull out two socks hoping to get a matching pair. Determine the probability:
  - Neither is white
  - You get one of each color
  - Both are white
  - Both are black
- Two dice are rolled. Determine the probability:
  - Sum is 10
  - Sum is greater than 10
  - You get doubles
  - You do not get doubles
- Three dice are rolled. Without writing out the entire sample space, determine the probability:
  - Sum is 5
  - Sum is greater than 15
- From a normal deck of 52 cards, a five card hand is drawn. Determine the probability for each:
  - One of the cards is an ace
  - You get four aces
  - You get two aces and three kings
  - None of the cards is an ace
  - You get at least one ace
- Two numbers are picked from the set  
What is the probability the:
  - Their sum is greater than 5
  - Their sum is less than 4
  - Their product is greater than 10
  - Their product is greater than 20
- There are three men and seven women applying for the same two jobs. What is the probability that at random:
  - Men are hired for the two jobs
  - One man and one woman are hired
  - Two women are hired



d. At least one woman is hired

### Probability WS 3 - More Probability Practice

Show your work – write down the set up and answer, when possible.

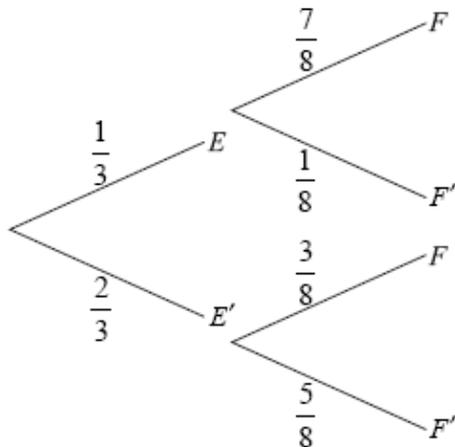
1. José travels to school on a bus. On any day, the probability that José will miss the bus is  $\frac{1}{3}$ .

If he misses his bus, the probability that he will be late for school is  $\frac{7}{8}$ .

If he does not miss his bus, the probability that he will be late is  $\frac{3}{8}$ .

Let  $E$  be the event “he misses his bus” and  $F$  the event “he is late for school”.  
The information above is shown on the following tree diagram.

- (a) Find  
(i)  $P(E \cap F)$ ;  
(ii)  $P(F)$ .



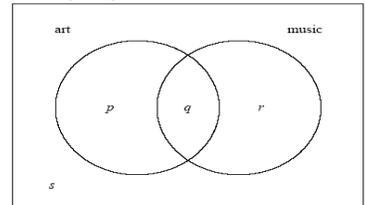
- (b) Find the probability that  
(i) José misses his bus  
(ii) José missed his bus

3. In a class of 100 boys, 55 boys play football and 75 boys play rugby. Each boy must play at least one sport from football and rugby.
- (a) (i) Find the number of boys who play both sports.  
(ii) Write down the number of boys who play only rugby.
- (b) One boy is selected at random.  
(i) Find the probability that he plays only one sport.  
(ii) Given that the boy selected plays only one sport, find the probability that he plays rugby.

Let  $A$  be the event that a boy plays football and  $B$  be the event that a boy plays rugby.

- (c) Explain why  $A$  and  $B$  are **not** mutually exclusive.  
(d) Show that  $A$  and  $B$  are **not** independent.

4. In a group of 16 students, 12 take art and 8 take music. One student takes neither art nor music. The Venn diagram below shows the events art and music. The values  $p$ ,  $q$ ,  $r$  and  $s$  represent numbers of students.



- (a) (i) Write down the value of  $s$ .  
(ii) Find the value of  $q$ .  
(iii) Write down the value of  $p$  and of  $r$ .
- (b) (i) A student is selected at random. Given that the student takes music, write down the probability the student takes art.  
(ii) **Hence**, show that taking music and taking art are **not** independent events.

- (c) Two students are selected at random, one after the other. Find the probability that the first student takes **only** music and the second student takes **only** art.

5. In any given season, a soccer team plays 65 % of their games at home. When the team plays at home, they win 83 % of their games. When they play away from home, they win 26 % of their games. The team plays one game.

- (a) Find the probability that the team wins the game.
- (b) If the team does not win the game, find the probability that the game was played at home.

6. In a survey, 100 students were asked “do you prefer to watch television or play sport?” Of the 46 boys in the survey, 33 said they would choose sport, while 29 girls made this choice.

	Boys	Girls	Total
Television			
Sport	33	29	
Total	46		100

By completing this table or otherwise, find the probability that

- (a) a student selected at random prefers to watch television;
- (b) a student prefers to watch television, given that the student is a boy.

7. There are 20 students in a classroom. Each student plays only one sport. The table below gives their sport and gender.

	Football	Tennis	Hockey
Female	5	3	3
Male	4	2	3

- (a) One student is selected at random.
- (i) Calculate the probability that the student is a male or is a tennis player.
- (ii) Given that the student selected is female, calculate the probability that the student does not play football.
- (b) Two students are selected at random. Calculate the probability that neither student plays football.

8. The table below shows the subjects studied by 210 students at a college.

	Year 1	Year 2	Totals
History	50	35	85
Science	15	30	45
Art	45	35	80
Totals	110	100	210

- (a) A student from the college is selected at random.
- Let  $A$  be the event the student studies Art.
- Let  $B$  be the event the student is in Year 2.
- (i) Find  $P(A)$ .
- (ii) Find the probability that the student is a Year 2 Art student.
- (iii) Are the events  $A$  and  $B$  independent? Justify your answer.
- (b) Given that a History student is selected at random, calculate the probability that the student is in Year 1.
- (c) Two students are selected at random from the college. Calculate the probability that

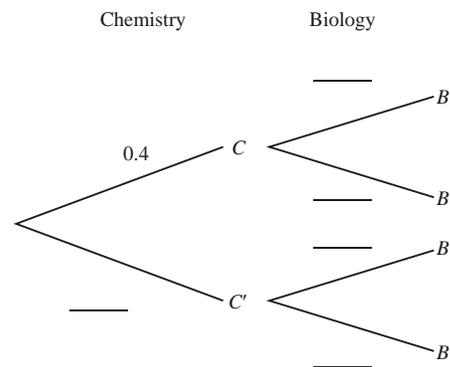
one student is in Year 1, and the other in Year 2.

9. In a class, 40 students take chemistry only, 30 take physics only, 20 take both chemistry and physics, and 60 take neither.
- Find the probability that a student takes physics given that the student takes chemistry.
  - Find the probability that a student takes physics given that the student does **not** take chemistry.
  - State whether the events “taking chemistry” and “taking physics” are mutually exclusive, independent, or neither. Justify your answer.

10. The events  $B$  and  $C$  are dependent, where  $C$  is the event “a student takes Chemistry”, and  $B$  is the event “a student takes Biology”. It is known that

$$P(C) = 0.4, P(B | C) = 0.6, P(B | C^c) = 0.5.$$

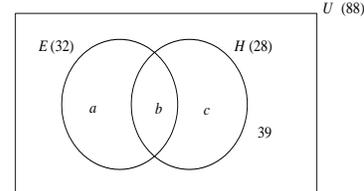
- Complete the following tree diagram.
- Calculate the probability that a student takes Biology.
- Given that a student takes Biology, what is the probability that the student takes Chemistry?



11. A box contains 22 red apples and 3 green apples. Three apples are selected at random, one after the other, without replacement.

- The first two apples are green. What is the probability that the third apple is red?
- What is the probability that exactly two of the three apples are red?

12. In a school of 88 boys, 32 study economics (E), 28 study history (H) and 39 do not study either subject. This information is represented in the following Venn diagram.



- Calculate the values  $a$ ,  $b$ ,  $c$ .
- A student is selected at random.
  - Calculate the probability that he studies **both** economics and history.
  - Given that he studies economics, calculate the probability that he does **not** study

history.

- (c) A group of three students is selected at random from the school.
- (i) Calculate the probability that none of these students studies economics.
  - (ii) Calculate the probability that at least one of these students studies economics.

13. Consider events  $A, B$  such that  $P(A) > 0, P(A) < 1, P(B) > 0,$  and  $P(B) < 1$ . In each of the situations (a), (b), (c) below state whether  $A$  and  $B$  are mutually exclusive (M); independent (I); neither (N).
- (a)  $P(A|B) = P(A)$
  - (b)  $P(A \cap B) = 0$
  - (c)  $P(A \cap B) = P(A)$

14. Dumisani is a student at IB World College.

The probability that he will be woken by his alarm clock is  $\frac{7}{8}$ .

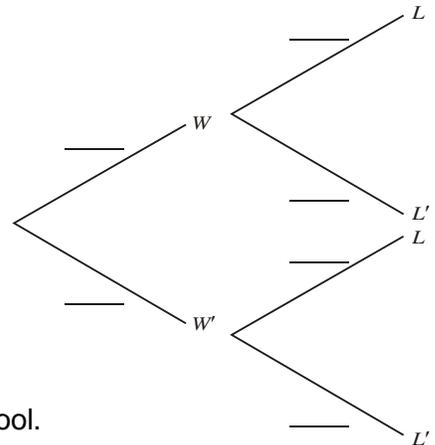
If he is woken by his alarm clock the probability he will be late for school is  $\frac{1}{4}$ .

If he is not woken by his alarm clock the probability he will be late for school is  $\frac{3}{5}$ .

Let  $W$  be the event "Dumisani is woken by his alarm clock".

Let  $L$  be the event "Dumisani is late for school".

- (a) Copy and complete the tree diagram below.



- (b) Calculate the probability that Dumisani will be late for school.
- (c) Given that Dumisani is late for school what is the probability that he was woken by his alarm clock?

## WS- Probability Review

1. A survey of residents in a certain town indicates 170 own a humidifier, 130 own a snow blower, and 80 own both a dehumidifier and snow blower. How many own either a dehumidifier or snow blower?
2. From a group of 10 people, in how many ways can we choose a chairperson, vice chair, treasurer and secretary, assuming one person cannot hold more than one position?
3. A teacher has 14 math books on her shelf. In how many ways can the books be arranged if there are 3 (identical) Algebra books, 4 (identical) Analysis books, 5 (identical) Statistics books, and two different Calculus books?
4. Find the number of ways eight people can be seated at an octagonal table (relative to each other).
5. A software company employs 9 sales reps and 8 technical reps. In how many ways can the company select 5 of these employees to send to a computer convention if at least 4 technical reps must attend?
6. An experiment consists of drawing a ball from a box that contains nine balls numbered 1 to 9. What is the probability that the ball number on the ball is even or divisible by 3?
7. Eight cards are drawn from a standard deck of cards. What is the probability that there are 5 face cards and 3 non-face cards?
8. In a law firm consisting of 20 lawyers, 9 are criminal lawyers, 6 are divorce lawyers and 4 can practice either criminal or divorce law. If a lawyer from the firm is chosen at random, what is the probability that he or she isn't a criminal or divorce lawyer?
9. A pair of dice is tossed. Find the odds that the sum of the dice facing up is 5.
10. Two marbles are drawn in succession out of a box that contains 3 green and 8 yellow marbles without replacement. Find the probability that exactly one green marble was drawn.
11. A little girl has 15 socks in her drawer, 7 are pink and 8 are purple. If she selects two socks at random, what is the probability that at least one of the socks is pink?
12. Each person in a group of students was identified by his/her hair color and then asked whether he or she preferred taking math classes in the morning or afternoon. The results are shown in the table below.

Class Time Preference	Blonde	Brunette	Redhead
Morning	45	25	5
Afternoon	40	20	20

- a. Find the probability that a randomly selected student from this group prefers afternoon math classes?
  - b. Find the probability that a randomly selected redhead from the group prefers afternoon math classes?
  - c. Are the events "prefers afternoon math classes" and "being a redhead" independent? Explain how you know.
13. A soccer team is to play two games in a tournament. The probability of winning the first game is 0.60. If the first game is won, the probability of winning the second game is 0.25. If the first game is lost, the probability of winning the second game is 0.15. What is the probability the team won its first game if we know the second game was lost?
  14. Each question on an 8-question multiple choice test has 5 possible answer choices, only one of which is correct. If a student randomly guesses on all 8 questions, find the probability the student gets more than 5 questions correct.
  15. Four coins are tossed. Find the probability that three are heads or two are tails.

## WS- Statistics Day 1: Probability Distributions

Do all work neatly on a separate piece of paper.

1. A student randomly guesses the answer for each question of a 6-question True/False exam. Let the random variable  $X$  be the number of questions answered correctly.
  - a. List the possible numerical outcomes (sample space) of  $X$ .
  - b. Find the probability of each outcome (Hint: it's binomial!)
  - c. Construct a histogram showing the probability distribution. Be sure to label your horizontal and vertical axis.
  - d. What is the probability that the student will get 4 or more questions correct?
  - e. What is the expected value of the number of questions the student will answer correctly?
2. You are going to roll a die 50 times.
  - a. What is the sample space?
  - b. What is the probability of each outcome?
  - c. Construct a histogram showing the probability distribution. Label your horizontal and vertical axes.
  - d. What is the expected value of a single dice roll? Will you ever actually roll that value?
  - e. Does your probability distribution represent theoretical or experimental probabilities?
3. Now you will construct an experimental probability distribution for rolling a die 50 times. Use your calculator MATH PRB randInt (1, 6) to simulate rolling a die one time. Repeat 50 rolls of the die and record the frequency of each outcome in a probability distribution table.

- a. Use your data to construct a frequency table and a probability distribution.

Outcome	1	2	3	4	5	6
Observed Frequency						
Probability						

- b. Construct a probability histogram for your data. Compare your experimental distribution to the theoretical distribution you drew for #2.
4. The following data show the salaries (in thousands of dollars) of 28 employees of a small company:  
30, 33, 35, 41, 42, 45, 45, 50, 52, 53, 55, 57, 60, 60, 61, 62, 63, 65, 67, 70, 71, 72, 73, 98, 105, 125, 150, 175
    - a. Construct a frequency table for the data. What class intervals will you use?
    - b. Draw a frequency histogram for the data. Include a title and label your axes.
    - c. Suppose that an adult is randomly selected from this company. What is the probability that s/he earns more than \$75,000?
  5. In the Pick 3 Lottery you choose a 3 digit number. If your number matches the number selected by the lottery board, you win \$500. Otherwise, you win nothing.
    - a. What are the possible numerical outcomes for your winnings?
    - b. What is the probability of each outcome?
    - c. Construct a histogram showing the probability distribution.
    - d. Find the expected value of your winnings.
    - e. A game is considered "fair" if the cost to play is equal to the expected value of the winnings. What would be a fair price to charge for a lottery ticket?

## Analysis CP

### WS- Statistics Day 2: Describing Distributions

Do all work neatly on a separate piece of paper.

1. Describe the shape of each of the distributions you constructed for Day 1.
2. The following data show the salaries (in thousands of dollars) of 28 employees of a small company:  
(Note: Data is the same as Day 1, # 4)  
30, 33, 35, 41, 42, 45, 45, 50, 52, 53, 55, 57, 60, 60, 61, 62, 63, 65, 67, 70, 71, 72, 73, 98, 105, 125, 150, 175
  - a. What are the mean and median salaries? Which one is a better measure of center? Why?
  - b. Find the range and IQR of the data. Which one is a better measure of the spread of the data? Why?
  - c. Find the standard deviation.
  - d. Use the 1.5 IQR rule to determine if the data has any outliers.
  - e. Draw a box plot of the data showing outliers (if present).

3. Use the formula  $s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$  to calculate the standard deviation of 0, 2, 5, 8, 10. What does this value tell you about the data?
4. The heights of a random sample of 19 men are recorded below:  
69.9, 71.8, 72.1, 73.1, 73.8, 70.6, 69.4, 69.6, 76.2, 71.8, 74.6, 66.9, 69.1, 66.7, 70.4, 71.8, 69.3, 72.3, 71.5
  - a. Use your calculator to construct a histogram of the data. Use class interval widths of 2 inches.
  - b. Describe the shape of the distribution.
  - c. Find the mean, median, and mode (class interval with the highest frequency). What do you notice about the three measures of center?
  - d. Construct a box plot for the data. Are there any outliers?
  - e. What interval of heights contains the middle 50% of the men?
  - f. What height interval contains the shortest 25% of the men? The tallest 25%?
5. A student randomly guesses the answer for each question of a 6-question True/False exam.  
(Note: You constructed the probability distribution on Day 1, Q #1)
  - a. Enter the numerical values of the sample space into L1 of your calculator. Enter the probability of each outcome into L2.
  - b. To have your calculator construct the probability histogram, press 2nd STAT PLOT 1: PLOT 1...On. Choose the histogram. Set Xlist: L1 and Freq: L2. (This allows the probabilities in L2 to be used as frequencies). Use the same window as you used for the graph yesterday. Is the calculator's graph the same as yours?
  - c. Follow these instructions carefully: Press STAT CALC 1-Var Stats ENTER L1, L2 ENTER . (The calculator will know to use the values in L2 as frequencies for L1). What is the mean of the distribution? Is it the same as the expected value you found yesterday?
  - d. Does the calculator report a sample standard deviation? For a probability distribution, the calculator provides the population standard deviation  $\sigma$ . What is this value?
  - e. Report Q1, Median, and Q3 for the distribution.
  - f. Use your calculator to construct a box plot.

WS- Statistics Day 3: Normal Distributions

Do all work neatly on a separate piece of paper.

**Table entries show the percent (P) of observations in a Normal distribution that are less than the z-score.**

z	-3.0	-2.5	-2.0	-1.5	-1.0	-.50	0	.50	1.0	1.5	2.0	2.5	3.0
P	0.1 3	0.6	2.3	6.7	15. 9	30. 9	50 %	69. 1	84. 1	93. 3	97. 7	99. 4	99.8 7

- The lengths of babies born at City Hospital last year were approximately Normally distributed with a mean of 20 inches and a standard deviation of 1 inch.
  - Draw a Normal curve showing the mean and  $\pm 1$ ,  $\pm 2$ ,  $\pm 3$  std. dev.
  - What percent of the babies born were longer than 20 inches?
  - What percent of the babies born were between 19 and 22 inches?
  - Would it be unusual for a baby to be more than 23 inches? Explain your answer.
  - What interval centered about the mean would contain 95% of all babies born?
- The amount of cola that a machine puts into soda cans is approximately Normally distributed with a mean of 355 mL and a standard deviation of 2 mL. Assume that the machine fills 2000 can a day.
  - Draw a Normal curve showing the mean and  $\pm 1$ ,  $\pm 2$ ,  $\pm 3$  std. dev.
  - About how many cans will contain more than 359 mL of cola?
  - About how many cans will contain between 353 and 357 mL of cola?
  - About how many cans will contain less than 352 mL of cola?
  - If the size of the can is 360 mL, about how many cans will overflow?
- Scores on the Weschler Adult Intelligence Scale are approximately Normally distributed with a mean of 110 and a standard deviation of 25.
  - Approximately what percent of adults will score below 98?
  - Approximately what percent of adults will score above 135?
  - MENSA is an elite organization that will only admit the top 2% of applicants. Approximately what score on the Weschler exam would an adult need to qualify for MENSA?
- A math teacher will be grading a test using a Normal curve. The mean score on the test was 80 and the standard deviation was 8.
  - What percent of the students will get A's if a score of 88 or above is an A?
  - What percent of students will get B's if a score of 80 to 88 is a B?
  - What percent of students will get C's if a score of 68 to 80 is a C?
  - The teacher only wants about 2% of his students to fail. What score should be the lowest passing (D) grade?
- When administered to 4th graders, a test of reading ability has a mean of 75 and standard deviation of 12. Sixth graders have a mean score of 85 with a standard deviation of 8 on a similar test. A young student scored 71 on the reading test as a 4th grader and 79 as a 6th grader.
  - Compute the z-score of the student as a 4th grader and a 6th grader.
  - Relative to his classmates, is his reading performance as measured by the test improving?
  - Do you need to know if the scores are Normally distributed to answer part a & b?

WS- Statistics Day 4: Review

Do all work neatly on a separate piece of paper.

1. A basket ball player who consistently makes 80% of his free throws attempts 5 free-throws during a game. Let the random variable X be the number of free throws made.
  - a. List the possible numerical outcomes for X.
  - b. Find the probability of each outcome.
  - c. Construct a histogram to display the probability distribution.
  - d. Describe the shape of the distribution.
  - e. What is the expected value of X. Explain what this number tells you.

2. The weight in pounds of children in a 4th grade class are given below:

64	71	57	67	74	65	59	62	67	72
84	60	68	72	91	77	69	76	88	99

- a. Use your calculator to construct a histogram of the data.
  - b. Describe the shape, center and spread of the data.
  - c. Use the 1.5 IQR rule to determine if there are any outliers.
  - d. Construct a box plot of the data.
  - e. What weight interval contains 50% of the students?
3. A survey of 100 randomly selected families in a small community asked how many pets lived in their household. The responses are given below.

Number of Pets	0	1	2	3	4	5	6	7
Frequency	14	22	20	18	10	8	6	2

- a. Identify the random variable and its possible numerical outcomes.
  - b. What is the probability that a family selected at random has 5 or more pets?
  - c. What is the expected value for the number of pets in a household in this community?
4. Use the formula  $s = \sqrt{\frac{\sum(x - \bar{x})^2}{n-1}}$  to calculate the standard deviation of 7, 8, 9, 10, 11, 12, 13. Explain what this value tells you about the data.
  5. The commute times for employees at a large company follow an approximately Normal distribution with a mean of 33 minutes and a standard deviation of 7 minutes.
    - a. Draw a Normal curve showing the mean and + 1, + 2, + 3 std. dev.
    - b. What percent of the company's employees commute less than 26 minutes?
    - c. What is the commute time of the middle 95% of the company's employees?
    - d. What percent of employees commute between 26 and 47 minutes?
    - e. Would you be surprised to hear that many employees have commute times over 50 minutes?
  6. The midterm scores in Professor Normal's statistic course had a mean of 84 and a standard deviation of 8. The final exam scores had a mean of 77 and a standard deviation of 11. Student A scored 72 on the midterm and 66 on the final. Has her test score relative to the other students in the class improved?

