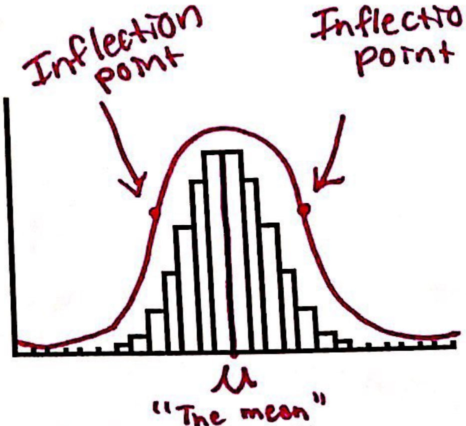
The Normal Distribution



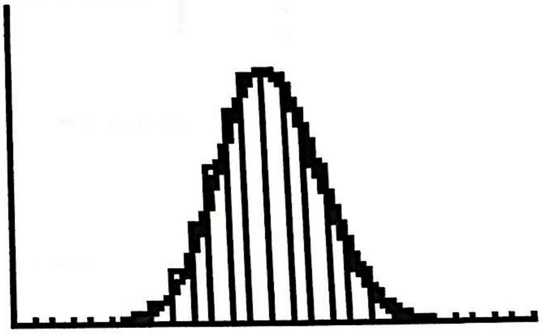
The probability distribution for a 25-question True/False quiz is shown. What is the shape of this distribution?
Symmetric, bell-shaped, Unimodal

Unimodal, symmetric distributions which follow a bell-shaped curve are called ***Normal Distributions***.

Notation for "mean"

A Normal distribution is centered at the mean, μ (mu). The ***inflection point*** of the curve is always 1 standard deviation, σ (sigma), away from the mean.

Notation for standard deviation



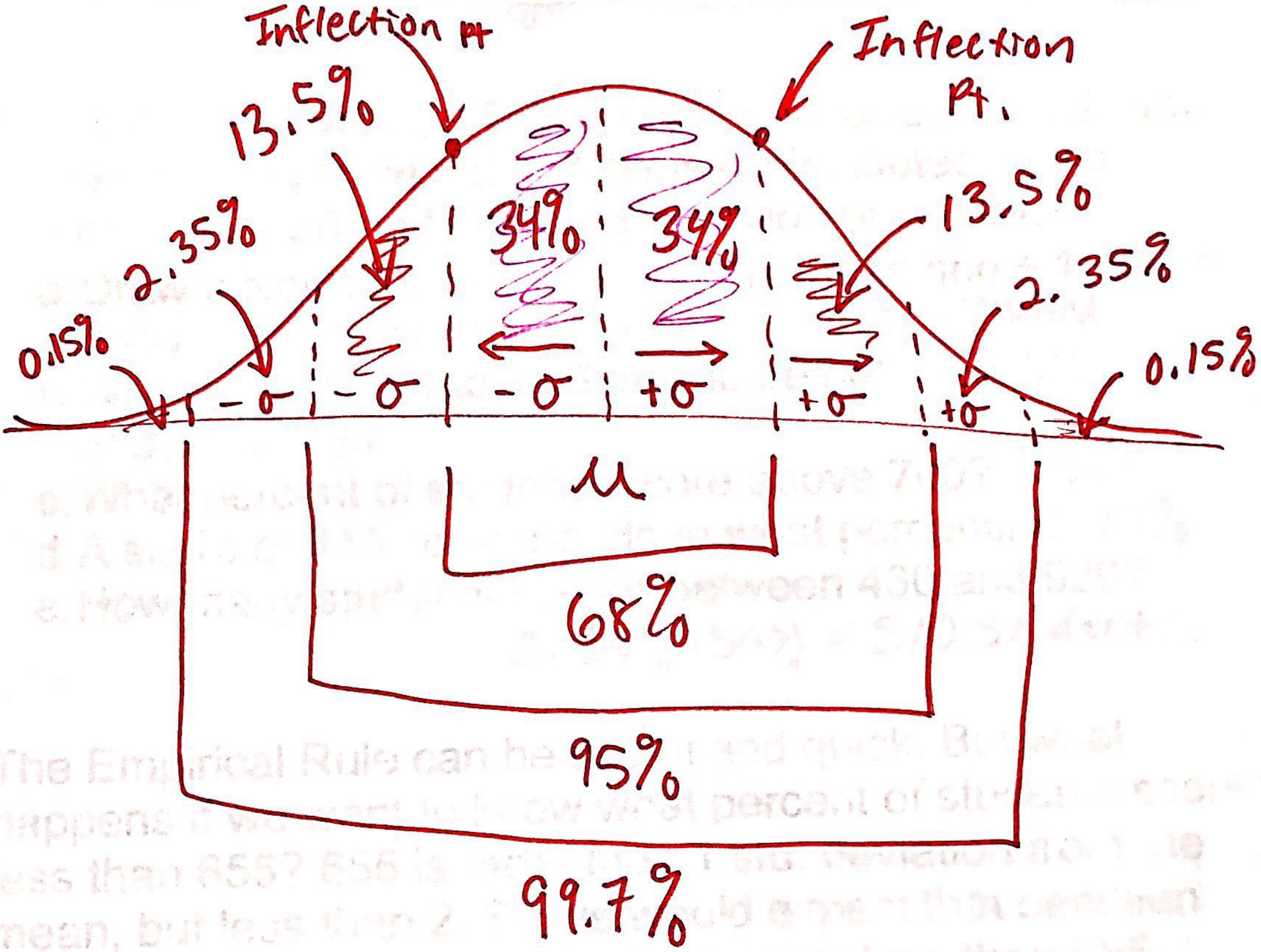
A Normal distribution is a probability distribution so the area underneath the curve represents probability. In particular, the ***Empirical Rule*** says that for a Normal distribution:

Know this

About 68% of the data is within ± 1 std. deviation of $\bar{x}$
About 95% of the data is within ± 2 std. deviation $\bar{x}$
About 99.7% of the data is within ± 3 std. deviation $\bar{x}$

★ See the normal distribution attached

Many examples of real-life data follow an *approximately* Normal distribution including heights, standardized test scores, repeated measurements of the same physical quantity, and True/False tests.


Inflection pt
Inflection pt.
13.5%
13.5%
34%
34%
2.35%
2.35%
0.15%
0.15%
-σ
-σ
-σ
+σ
+σ
+σ
μ
68%
95%
99.7%

Example: The Math SAT scores of 1500 randomly selected students are approximately Normally distributed with a mean of $\mu = 520$ and standard deviation of $\sigma = 90$.

a. Draw a Normal curve showing the mean and ± 1, ± 2, ± 3 std. dev. See the Normal Curve attached.

b. What percent of the students score between 430 and 610? 68%

c. What percent of students score above 700? 2.5%

d. A score of 340 corresponds to what percentile? 95%

e. How many students scored between 430 and 520?
0.34(1500) = 510 students

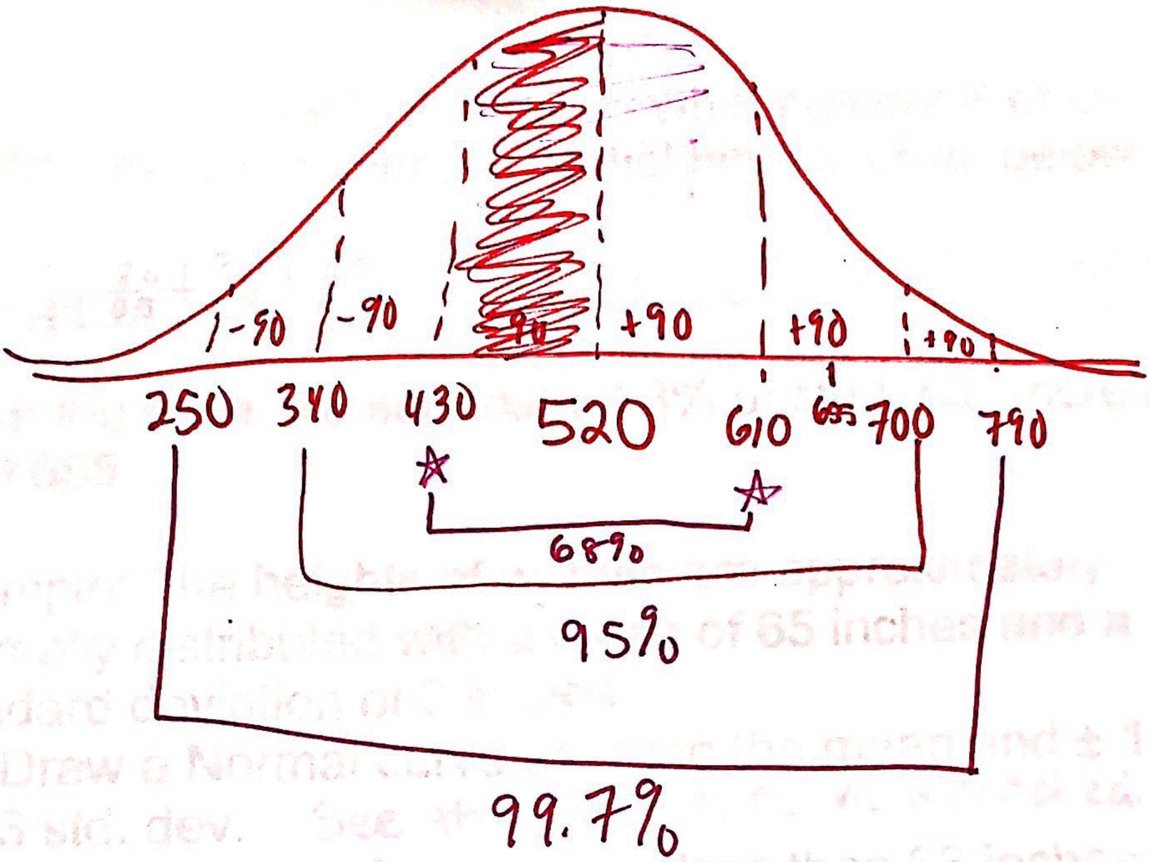
The Empirical Rule can be useful and quick. But what happens if we want to know what percent of students score less than 655? 655 is more than 1 std. deviation from the mean, but less than 2. So we would expect that between 84% and 97.5% of the students to score less than 655.

We'll start by calculating a ***z-score***, which tells us how many standard deviations the value of 655 is away from the mean.

$$z = \frac{\text{value - mean}}{\text{standard deviation}} = \frac{x - \mu}{\sigma}$$

$$z = \frac{655 - 520}{90} = \boxed{1.5}$$ standard deviations

z-score


-90
-90
-90
+90
+90
+90
250
340
430
520
610
655
700
790
68%
95%
99.7%

Tables are available that tell us what percent P of the data *is less than* the z-score. A partial table is given below:

for "d" for "e"

z	-3.0	-2.5	-2.0	-1.5	-1.0	-.50	0	.50	1.0	1.5	2.0	2.5	3.0
P	0.13	0.6	2.3	6.7	15.9	30.9	50%	69.1	84.1	93.3	97.7	99.4	99.87

From the table, we see that 93.3% of students score less than 655.

Example: The heights of women are approximately Normally distributed with a mean of 65 inches and a standard deviation of 2 inches.

a. Draw a Normal curve showing the mean and ± 1, ± 2, ± 3 std. dev. See the normal curve attached.
b. What percent of women are less than 63 inches tall? 16%
c. What percent of women are less than 67 inches tall? 84%
d. What percent of women are less than 62 inches tall? 6.7%
e. What percent of women are greater than 66 inches tall? 30.9%

d) $\frac{62-65}{2} = -1.5 \rightarrow \boxed{6.7\%}$ Use the table above

e) $\frac{66-67}{2} = -0.5 \rightarrow \boxed{30.9\%}$

Z-scores can help us compare data from two different distributions.

Example: Chris scored 720 on his Math SAT. Jimmy scored a 31 on the ACT test. Who performed better? (Assume the SAT has a mean of 520 and std. dev. of 90 and the ACT has a mean of 18 and std. dev. of 6).

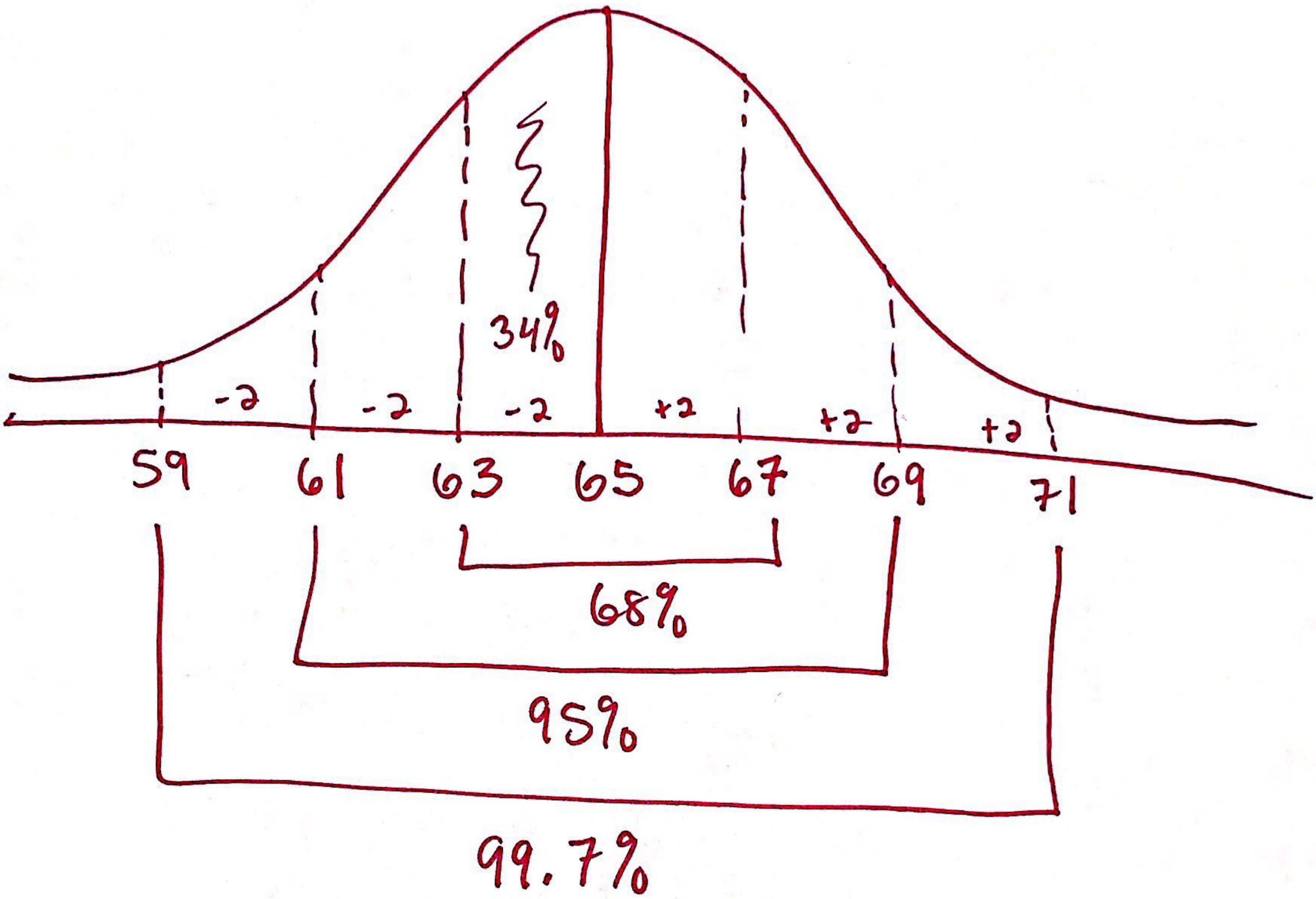
Chris:

$$z = \frac{720-520}{90} = 2.22$$

Jimmy:

$$z = \frac{31-18}{6} = 2.1$$

Chris scored higher



b) $50\% - 34\% = 16\%$

c) $50\% + 34\% = 84\%$