

Practice Test- Critical Points, Derivatives, Rational Functions

1. Find the extrema of  $f(x) = x^7 - 3x^2 + x + 5$  using a calculator. You do NOT need to show work, but you do need to indicate the method used (trace function, table, or min/max function).
2. The function  $f(x) = -x^3 - 6x^2 - 12x - 7$  has a critical point when  $x = -2$ . Identify the point as a maximum, a minimum, or a point of inflection, and state its coordinates.
3. Suppose that during an experiment you launch a toy rocket straight upward from a height of 6 inches with an initial velocity of 32 feet per second. The height at any time  $t$  can be modeled by the function  $s(t) = -16t^2 + 32t + 0.5$  where  $s(t)$  is measured in feet and  $t$  is measured in seconds. Use the derivative to find the maximum height obtained by the rocket before it begins to fall.
4. Find derivative of  $f(x) = 2x^2 + 5x$  using the definition of the derivative.
5. Find derivative of the following.
  - a.  $f(x) = 5x^3 - 2x^2 - x + 3$
  - b.  $f(x) = (2x + 7)(3x - 8)$
6. Acceleration is the rate at which the velocity of a moving object changes. The velocity in meters per second of a particle moving along a straight line is given by the function  $v(t) = 3t^2 - 6t + 5$  where  $t$  is the time in seconds. Find the acceleration of the particle in meters per second squared after 5 seconds. (*Hint: Acceleration is the derivative of velocity.*)
7. Graph by hand:
  - a.  $y = \frac{3}{x+1} - 2$
  - b.  $y = \frac{1}{x-3} + 4$
8. Identify discontinuities of rational functions (holes, vertical asymptote, horizontal asymptote).
  - a.  $f(x) = \frac{(x-1)(x+4)}{x(x-2)(x+4)}$
  - b.  $f(x) = \frac{(x+1)(-7x+5)}{(x-1)(3x-2)}$

Answer key:

1. relative maximum at (.167, 5.08), relative minimum (.93, 3.94)
2. point of inflection at (-2, 1)
3. 16.5 feet
4.  $f'(x) = 4x+5$
5. a.  $f'(x) = 15x^2 - 4x - 1$
- b.  $f'(x) = 12x + 5$
6. 24 m/s<sup>2</sup>
7. Graphs
8. a. hole at -4, vertical asymptote at  $x = 2$  and  $x = 0$ ; horizontal asymptote at  $y = 0$
- b. no hole, vertical asymptote at  $x = 2/3$ , horizontal asymptote is  $y = -7/3$